Quantifying EPR: the resource theory of nonclassicality of common-cause assemblages

Beata Zjawin[1], David Schmid[1], Matty J Hoban[2], Ana Belén Sainz[1]

[1] *International Centre for Theory of Quantum Technologies, University of Gdańsk, 80-308 Gdańsk, Poland*

[2] *Cambridge Quantum Computing Ltd, 9a Bridge Street, Cambridge, CB2 1UB, United Kingdom*

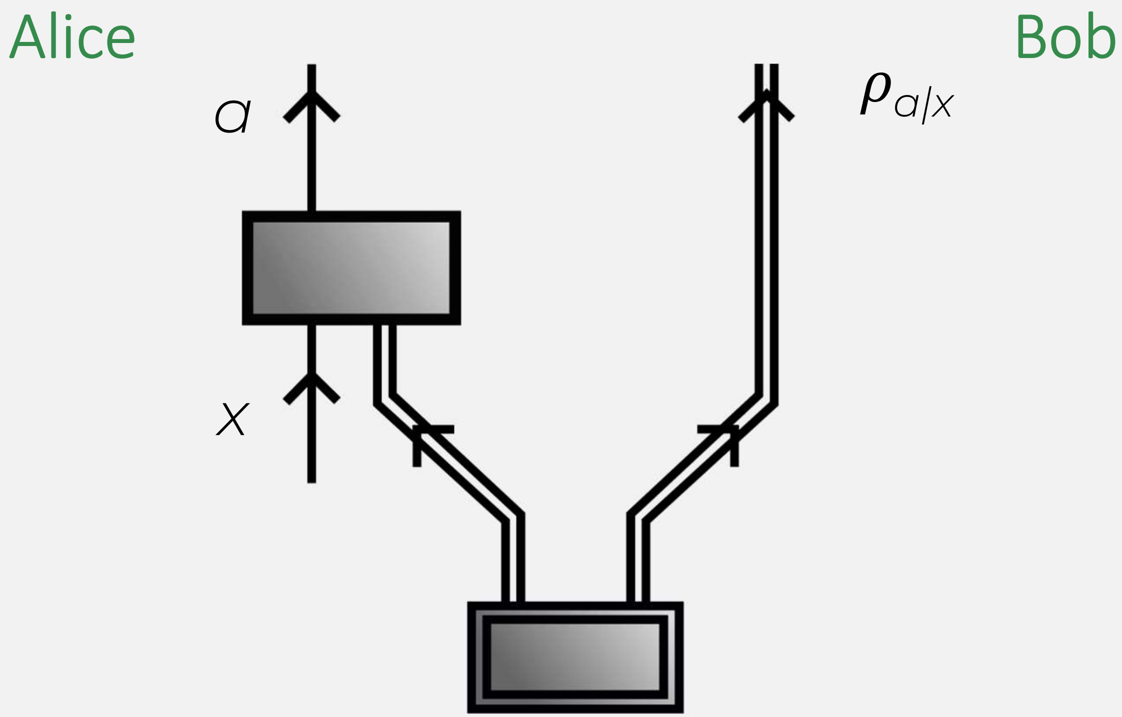
beatazjawin@gmail.com

University of Gdańsk

INTERNATIONAL CENTRE FOR THEORY OF QUANTUM TECHNOLOGIES

Standard EPR scenario

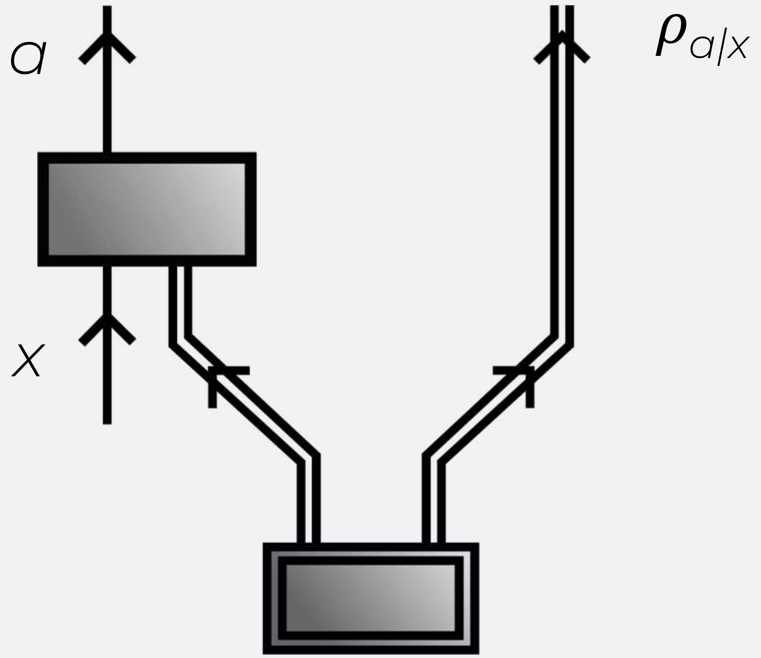
A form of **nonclassical correlations** that arise when one considers measurements performed on half of a bipartite system prepared on an entangled state.



Quantum assemblage

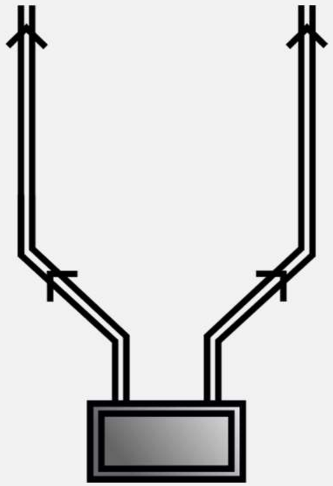
$\boldsymbol{\Sigma}_{A|X} = \{\boldsymbol{\sigma}_{a|x}\}_{\{a,x\}}$

$\boldsymbol{\sigma}_{a|x} = p(a|x)\,\boldsymbol{\rho}_{a|x}$

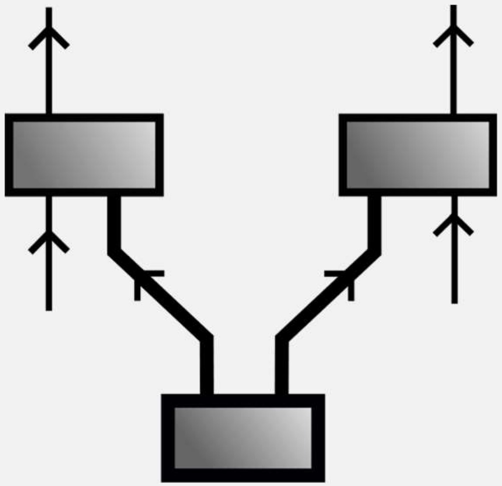


Multiple applications in quantum information protocols

Common-cause resources



Entanglement
arXiv:2004.09194



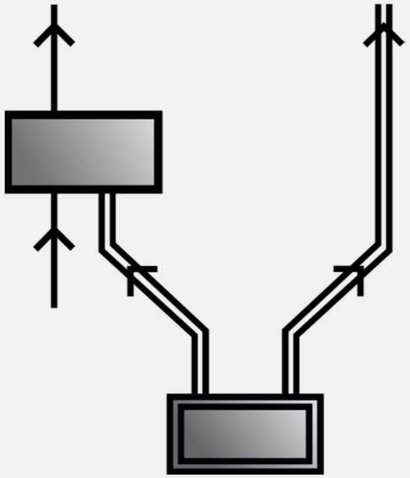
Bell scenarios
arXiv:1903.06311

Other common-cause processes:
arXiv:1909.04065

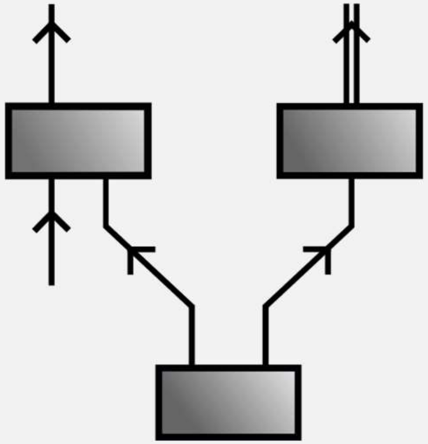
The resource theory of assemblages

Free operations: local operations and shared randomness

General assemblage:



Free assemblage:



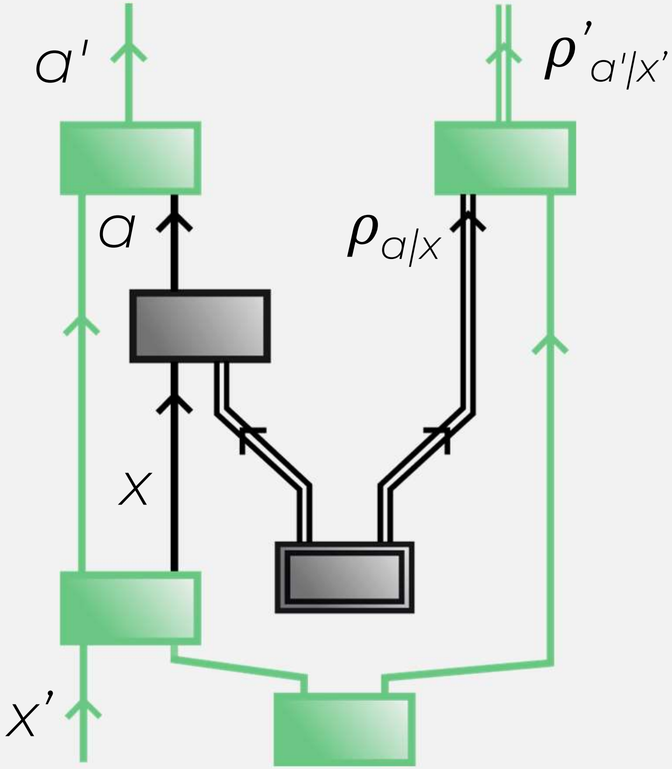
$$\boldsymbol{\sigma}_{a|x} = \Sigma_\lambda\, p(a\,|x\,\lambda)\, \boldsymbol{\sigma}_\lambda$$

(unsteerable assemblage)

LOSR operations

$\boldsymbol{\Sigma}_{A|X} = \{\boldsymbol{\sigma}_{a|x}\}_{\{a,x\}}$

$\boldsymbol{\sigma}_{a|x} = p(a|x)\, \boldsymbol{\rho}_{a|x}$



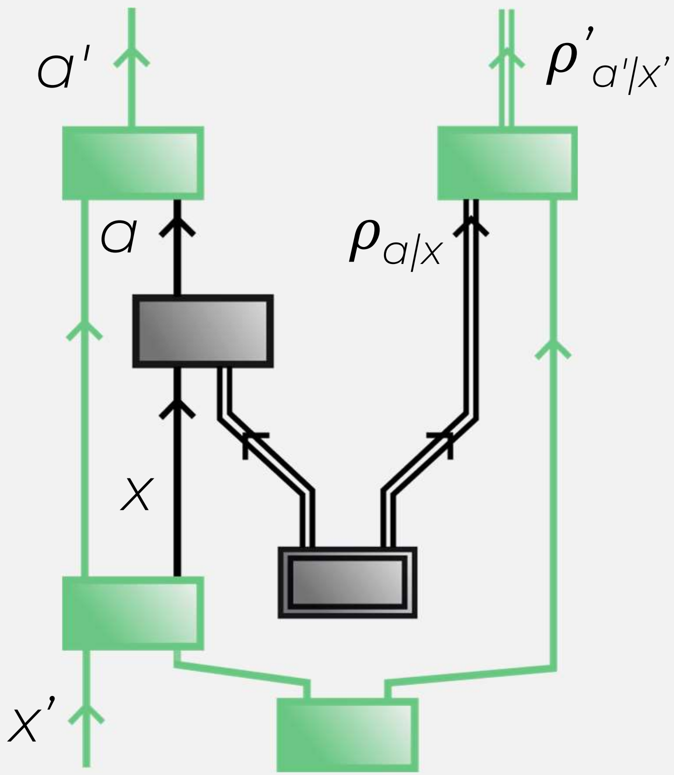
$\boldsymbol{\Sigma}_{A'|X'} = \{\boldsymbol{\sigma}_{a'|x'}\}_{\{a',x'\}}$

$\boldsymbol{\sigma}'_{a'|x'} = p(a'|x')\, \boldsymbol{\rho}'_{a'|x'}$

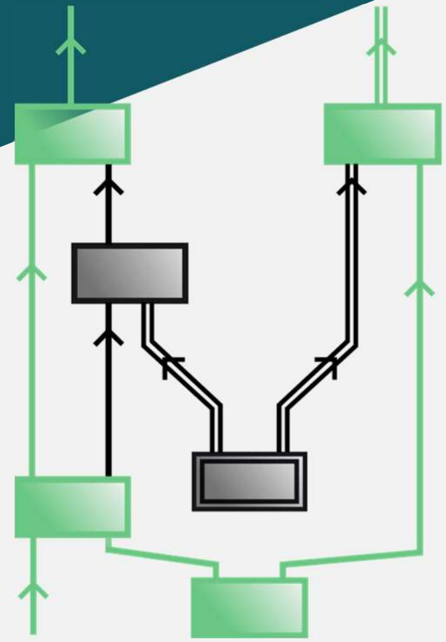
Properties of the pre-order

Assemblage conversion under LOSR

One assemblage is said to be *more nonclassical* than another if it can be freely converted to the latter.



Assemblage conversion under LOSR can be tested using a single instance of a semidefinite program



$$
\begin{aligned}
\textit{given} \quad & \{\sigma_{a|x}\}_{a,x}\,,\ \{\sigma_{a'|x'}\}_{a',x'}\,,\ \{D(a'|a,x',\lambda)\}_{\lambda,a',a,x'}\,,\ \{D(x|x',\lambda)\}_{\lambda,x,x'} \\
\textit{find} \quad & \{W_\lambda\}_\lambda \\
s.t. \quad & \begin{cases}
W_\lambda \geq 0\,, \\
\mathrm{tr}_{B'}\{W_\lambda\} \propto \frac{1}{d}\mathbb{I}_B \quad \forall\lambda\,, \\
\sum_\lambda \mathrm{tr}_{B'}\{W_\lambda\} = \frac{1}{d}\mathbb{I}_B\,, \\
\sigma_{a'|x'} = \sum_\lambda \sum_{a,x} D(a'|a,x',\lambda)\, D(x|x',\lambda)\, d\,\mathrm{tr}_B\left\{W_\lambda\, \mathbb{I}_{B'} \otimes \sigma^T_{a|x}\right\}\,.
\end{cases}
\end{aligned}
$$

Structure of the pre-order

We find an infinite family of incomparable resources (none of them can be converted into any other).

$$\sigma^{\theta}_{a|x} = \mathrm{tr}_{\mathrm{A}} \left\{ \widetilde{M}_{a|x} \otimes \mathbb{I} \left|\theta\right\rangle \left\langle\theta\right| \right\} ,$$

$$\left|\theta\right\rangle = \cos\theta \left|00\right\rangle + \sin\theta \left|11\right\rangle ,$$

$$\widetilde{M}_{a|0} = \frac{\mathbb{I} + (-1)^a \sigma_z}{2} , \quad \widetilde{M}_{a|1} = \frac{\mathbb{I} + (-1)^a \sigma_x}{2} .$$

We confirm this result with our SDP and analytically using EPR monotones that we develop.

Structure of the pre-order

We find an infinite family of incomparable resources (none of them can be converted into any other).

$$\sigma^{\theta}_{a|x} = \mathrm{tr}_{\mathrm{A}} \left\{ \widetilde{M}_{a|x} \otimes \mathbb{I} \, |\theta\rangle \langle\theta| \right\} ,$$

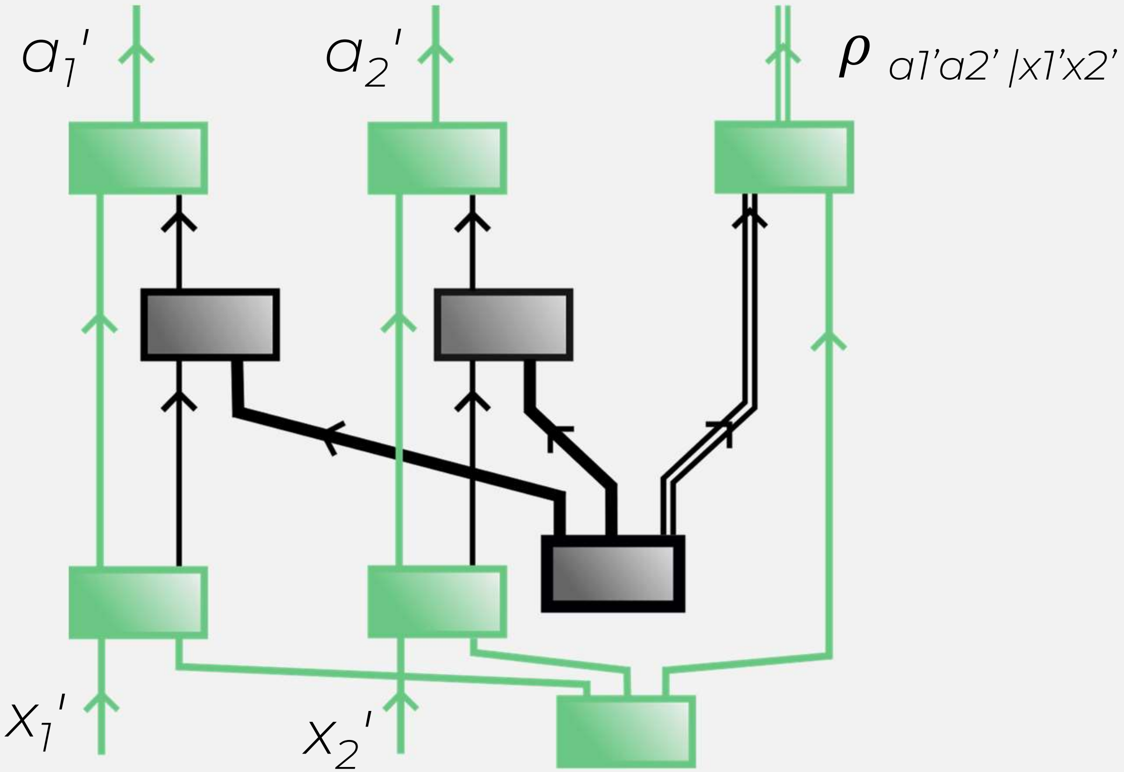
$$|\theta\rangle = \cos\theta \, |00\rangle + \sin\theta \, |11\rangle \, ,$$

$$\widetilde{M}_{a|0} = \frac{\mathbb{I} + (-1)^a \sigma_z}{2} , \quad \widetilde{M}_{a|1} = \frac{\mathbb{I} + (-1)^a \sigma_x}{2} .$$

Distillation with LOSR?

Multipartite scenario

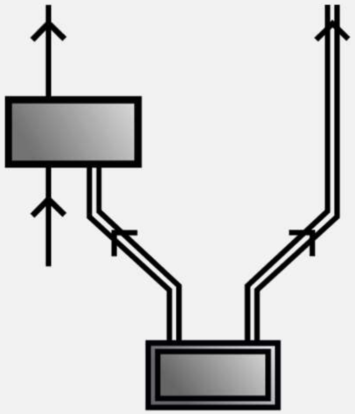
Multipartite EPR scenarios



- Assemblage conversion under LOSR can be tested using a single instance of a semidefinite program
- Family of incomparable resources

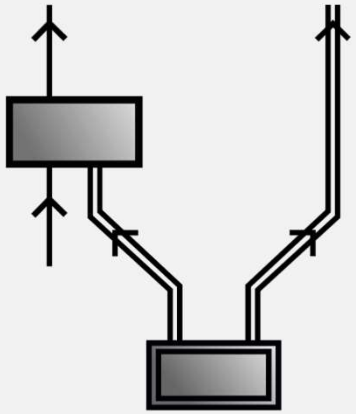
Bipartite generalizations

Bipartite generalizations

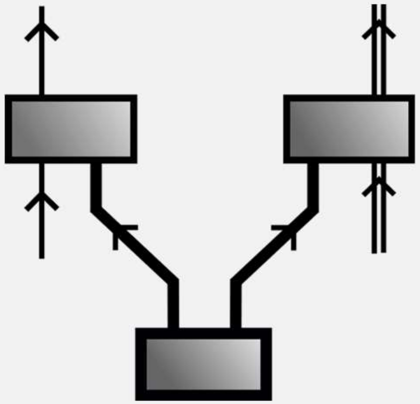


EPR

Bipartite generalizations

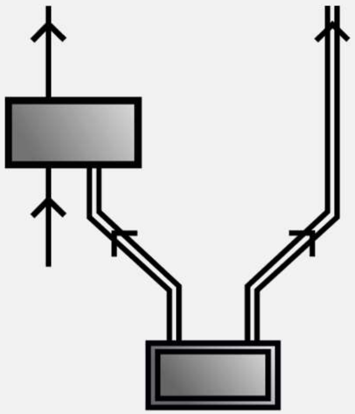


EPR

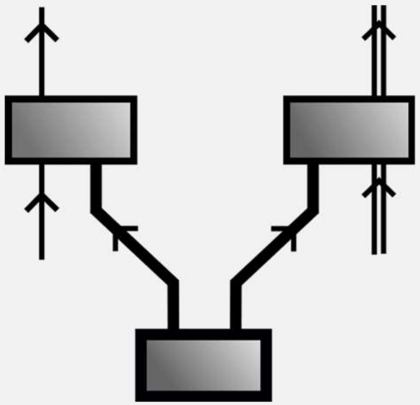


Channel

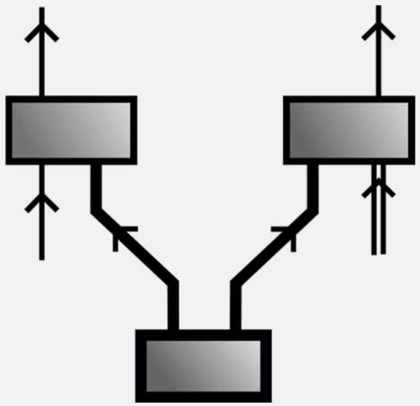
Bipartite generalizations



EPR

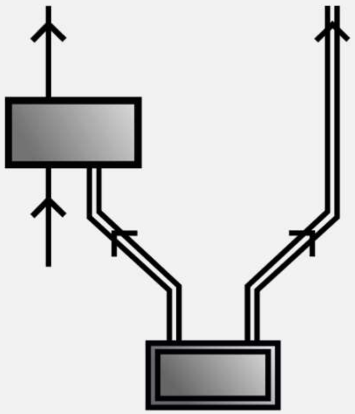


Channel

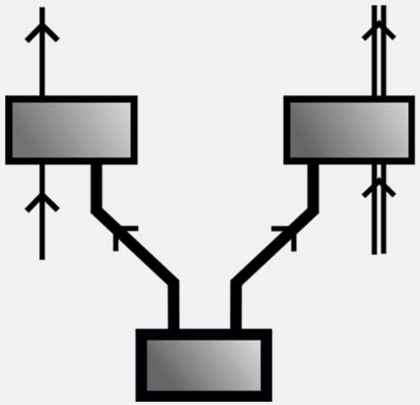


MDI

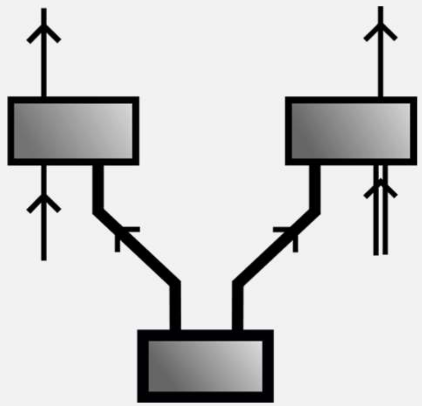
Bipartite generalizations



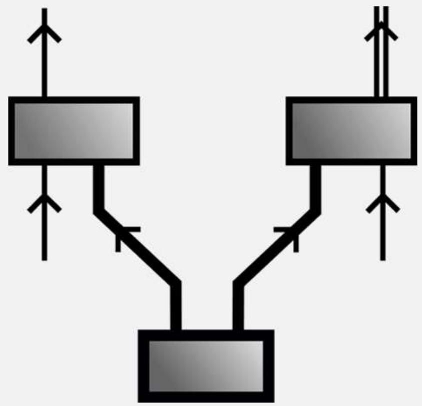
EPR



Channel

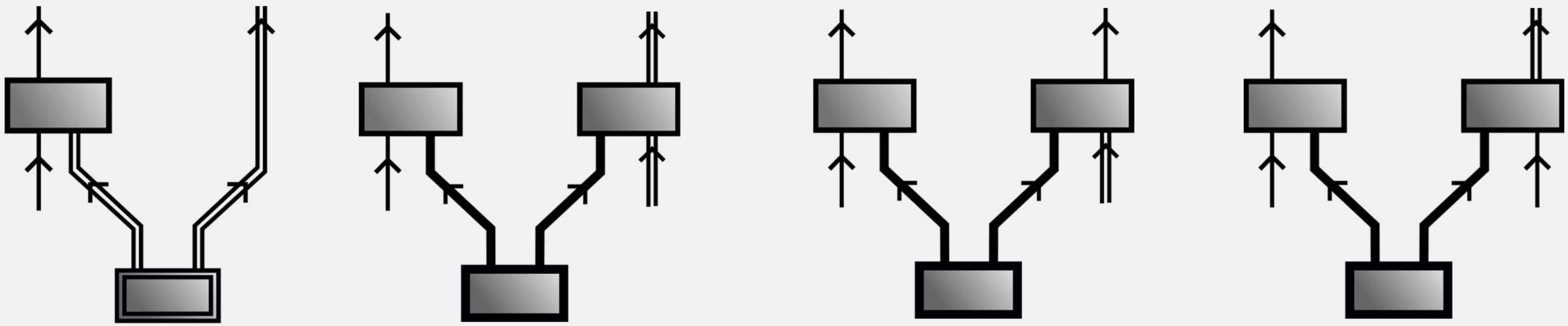


MDI



BwI

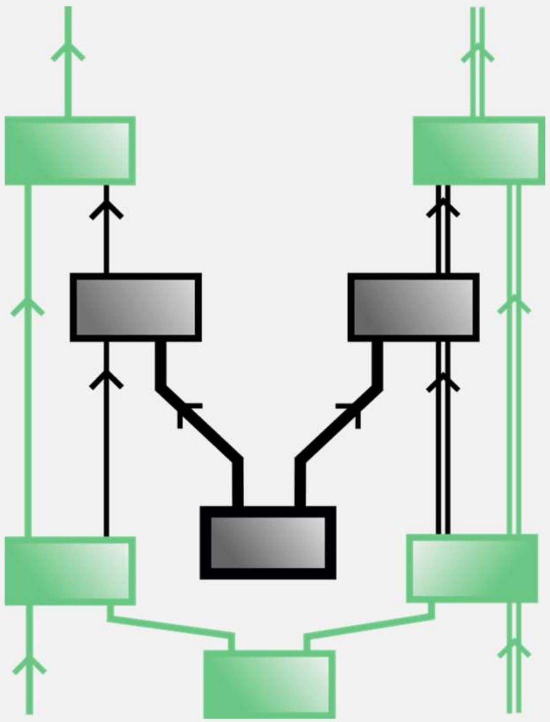
Bipartite generalizations



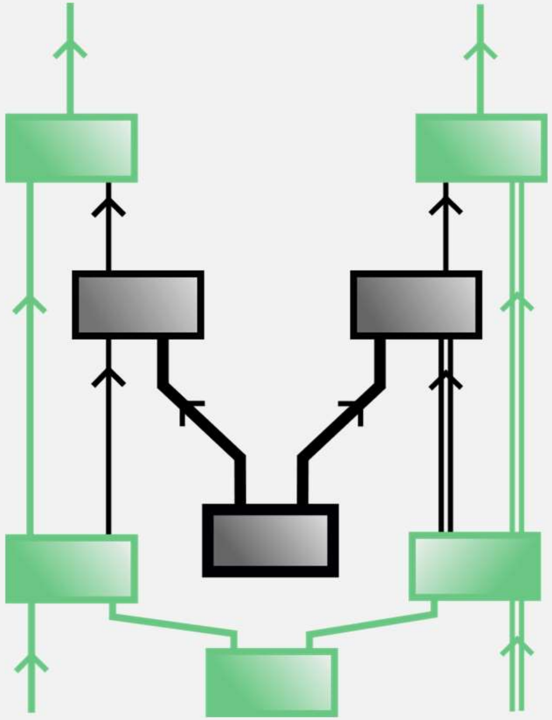
EPR Channel MDI BwI

Free resources = classical common cause

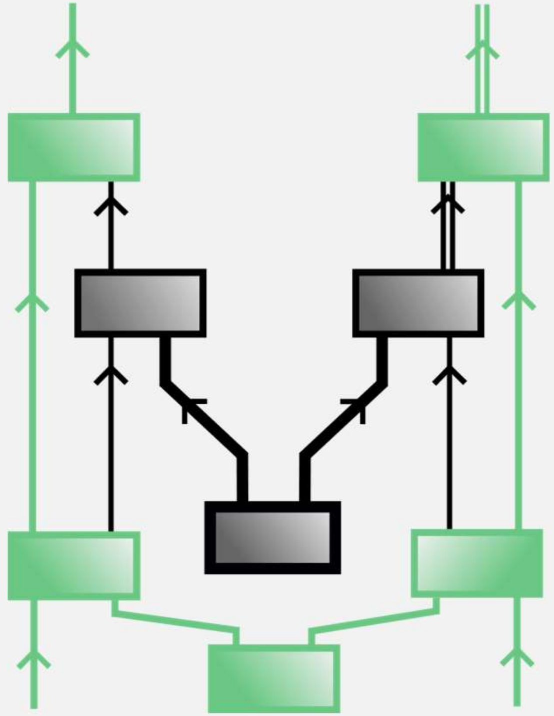
LOSR transformations



Channel



MDI

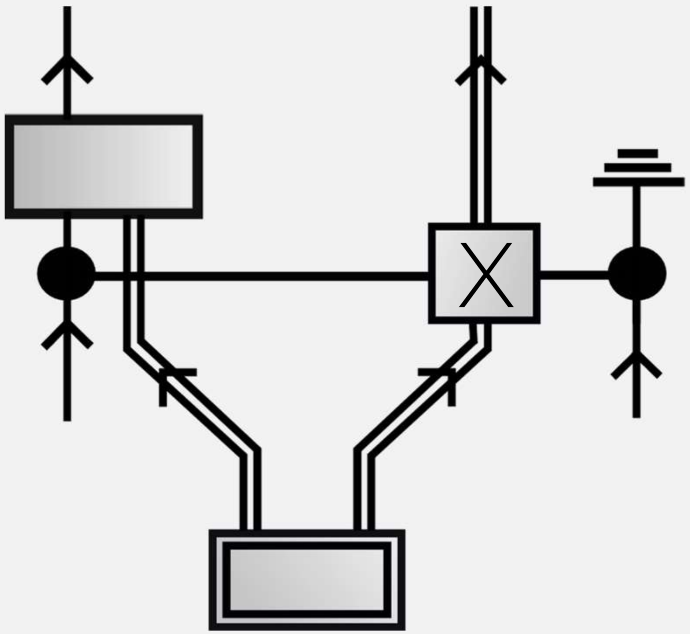


BwI

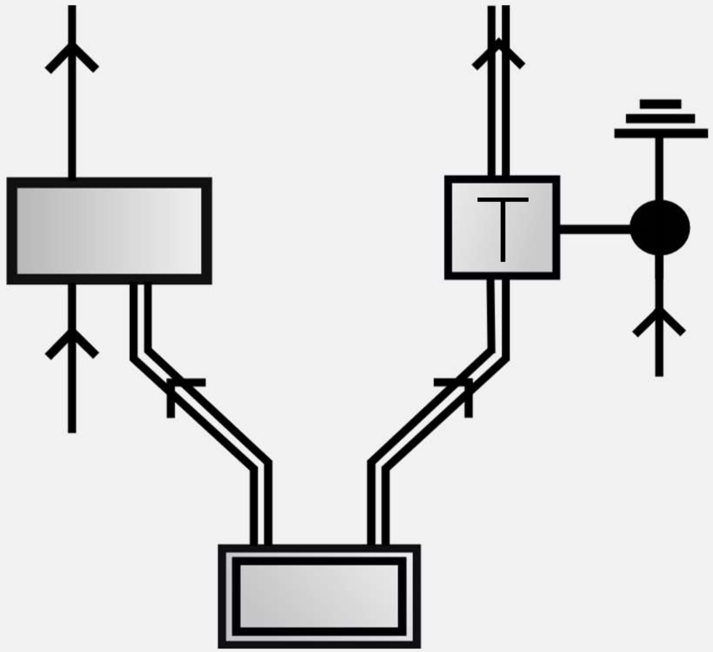
Assemblage conversion under LOSR can be tested using a single instance of a semidefinite program

Postquantum Bob-with-input assemblages

PR-box assemblage

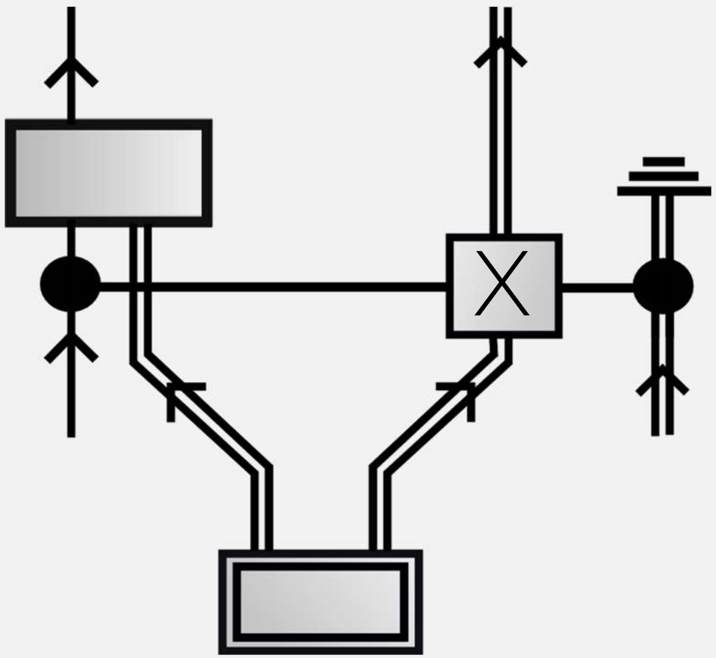


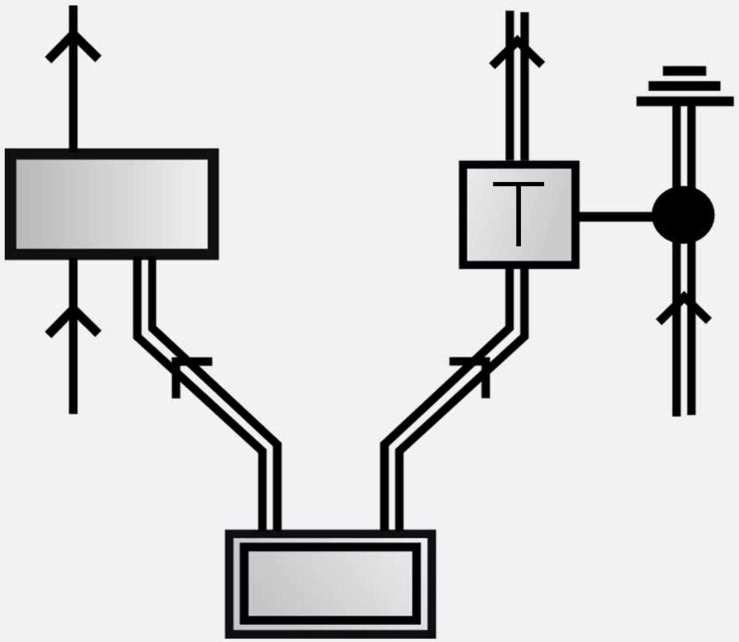
PTP assemblage



- Incomparable under LOSR
  - PR -> PTP under LOSE

Postquantum channel assemblages





Measurement-device-independent scenario

A hierarchy of SDPs to test membership to the quantum set

Final remarks

Resource theory of common-cause assemblages

(standard bipartite & multipartite, channel, Bob-with-input, measurement-device-independent)

arXiv:2111.10244

Resource theory of common-cause assemblages

(standard bipartite & multipartite, channel,
Bob-with-input, measurement-device-independent)

Unified notion of common-cause resources

(EPR, Bell,entanglement)

arXiv:2111.10244

Resource theory of common-cause assemblages

(standard bipartite & multipartite, channel, Bob-with-input, measurement-device-independent)

Unified notion of common-cause resources

(EPR, Bell,entanglement)

Testing conversions is numerically tractable

(interesting properties of the pre-order)

Thank you!

beatazjawin@gmail.com

arXiv: 2111.10244, 2209.10177

Comparison to prior work

Rodrigo Gallego and Leandro Aolita. *Resource theory of steering*. Physical Review X, 5 (4):041008, 2015
https://doi.org/10.1103/PhysRevX.5.041008

Free operations: Stochastic local operations and one-way classical communication from Bob to Alice (S-1W-LOCC)

Differences:

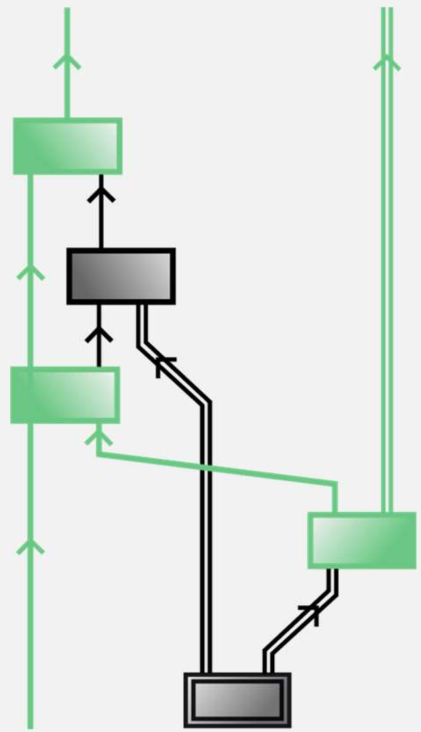
- Different pre-orders

Conceptual advantages:

- Clear physical motivation
- Unification of every type of nonclassical correlation in Bell-like scenarios

Technical advantages:

- simpler to characterize and study
- direct generalizations: multipartite EPR scenarios, Bob-with-input EPR scenarios and channel EPR scenarios



Structure of the pre-order

Family of assemblages indexed by two parameters:

$$\mathsf{S} = \left\{ \mathbf{\Sigma}^{\theta,p}_{\mathbb{A}|\mathbb{X}} \,\middle|\, \theta \in (0, \pi/4] \,, p \in [0,1] \right\} \,,$$

$$\text{where} \quad \mathbf{\Sigma}^{\theta,p}_{\mathbb{A}|\mathbb{X}} = \left\{ p\, \sigma^{\theta}_{a|x} + (1-p)\, \frac{\mathbb{I}}{4} \right\}_{a\in\mathbb{A},\, x\in\mathbb{X}} \,,$$

$$\text{with} \quad \sigma^{\theta}_{a|x} = \mathrm{tr}_{\mathrm{A}} \left\{ \widetilde{M}_{a|x} \otimes \mathbb{I} \,|\theta\rangle \langle\theta| \right\} \,,$$

$$|\theta\rangle = \cos\theta \,|00\rangle + \sin\theta \,|11\rangle \,,$$

$$\widetilde{M}_{a|0} = \frac{\mathbb{I} + (-1)^a \sigma_z}{2} \,, \quad \widetilde{M}_{a|1} = \frac{\mathbb{I} + (-1)^a \sigma_x}{2} \,.$$