RESOURCE ENGINES

Quantum resources: from mathematical foundations to operational characterisation

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From Heat Engines to Resource Engines

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Motivation:

- Resource engines – provide a natural way of fusing various resource theories (in the spirit of multi-resource theories).

  [C. Sparaciari, L. Del Rio, C. M. Scandolo, P. Faist, and J. Oppenheim, **Quantum** 4, 259, **2020**.]

From Heat Engines to Resource Engines

Alice	**Bob**
$\mathcal{F}_A$ – free operations F_A – free states	$\mathcal{F}_B$ – free operations F_B – free states

communication rounds (**strokes**)

$\rho \in F_A$

$\phi_i \in \mathcal{F}_A$ $\qquad\qquad$ $\psi_i \in \mathcal{F}_B$

ρ

$\searrow$

$\psi_1(\rho)$

$\swarrow$

$\phi_1\left(\psi_1(\rho)\right)$

$\searrow$

$\vdots$

Research Questions

Can we achieve all possible final states starting from free states?

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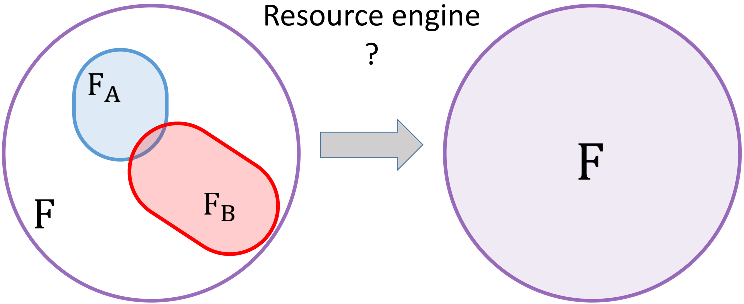

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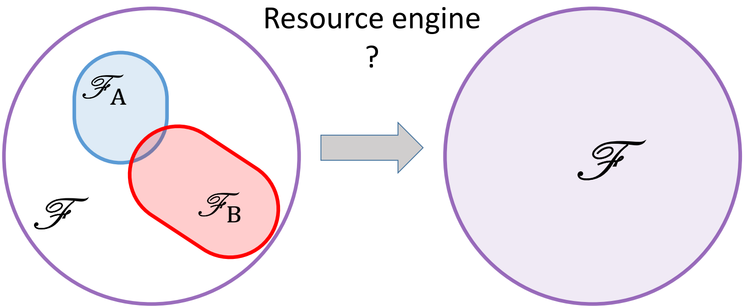

Research Questions

Can we bound the number of strokes
needed to obtain the above,
and thus study the equivalents of engine's power and
efficiency?

Exemplary Models of Resource Engines

1. Coherence engines.	2. Fixed point constraints.
Agents constrained to performing unitaries diagonal in 2 different bases.	Thermodynamic scenario with access to hot and cold baths, but with agents not allowed to use any ancillary systems.
for simplicity: theory restricted to pure states	for simplicity: theory restricted to incoherent states
related to: the problem of controllability by 2 different incommensurable Hamiltonians	

Coherence Engines

Notation and Assumptions

- $\mathcal{U}_n(\mathbb{C})$ – a group of unitary matrices of order $n \geqslant 2$ over $\mathbb{C}$.
- $\mathcal{DU}_n(\mathbb{C})$ – a subgroup of $\mathcal{U}_n(\mathbb{C})$ consisting of diagonal matrices.

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- Sets of **free operations**:

  $\mathcal{F}_A = \mathcal{DU}_n(\mathbb{C})$ and

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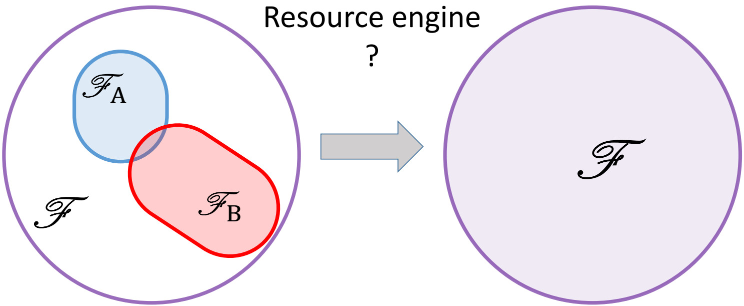
- For $U \in \mathcal{U}_n(\mathbb{C})$, define $P_U = (p_{ij})_{i,j=1}^n$ by

$$p_{ij} = \begin{cases} 0 & \text{for } u_{ij} = 0 \\ 1 & \text{for } u_{ij} \neq 0 \end{cases}.$$

Condition on Generating All Operations



Condition on Generating All Operations

(H1) There exist a constant $N \in \mathbb{N}$ and matrices $D_1, \ldots, D_{2N} \in \mathcal{DU}_n(\mathbb{C})$ such that

$$D_1 \left(U^\dagger D_2 U\right) D_3 \left(U^\dagger D_4 U\right) \ldots D_{2N-1} \left(U^\dagger D_{2N} U\right)$$

is a matrix with **all non-zero entries**.

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(H2) There exists a constant $N \in \mathbb{N}$ such that

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Lemma

Hypothesis (H2) and (H1) are ***equivalent****.*

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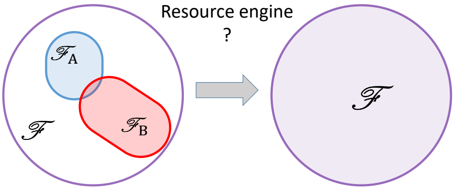

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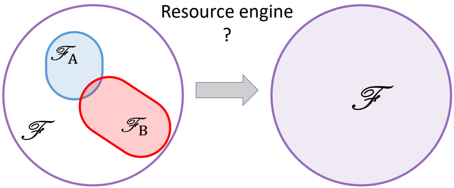

Theorem

IF U (appearing in the definition of $\mathcal{F}_B$) satisfies either (H1) or (H2), THEN any unitary matrix can be written as a product comprised of N matrices from $\mathcal{F}_A$ and N matrices from $\mathcal{F}_B$.

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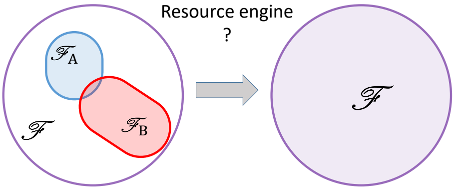

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[Z. Borevich and S. Krupetskij, J. Sov. Math. 17, 1718 (**1981**)]

Examples

Example (negative)

There exist unitary matrices which **do not satisfy condition (H2)**, e.g.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}.$$

Link to the Theory of Markov Chains

Theorem

Let X be an ***irreducible*** *and* ***aperiodic Markov chain*** *with finite state space and transition matrix Π. THEN there* ***exists a finite constant*** *M such that for all $m \geqslant M$*

$$\Pi_{ij}(m) > 0 \quad \textit{for all states} \quad i, j \in \Sigma,$$

meaning that $\Pi(m)$ ***is a matrix with all non-zero entries****, and so any two states are communicating.*

$$\Pi(0) = \mathbb{1}, \;\; \Pi(1) = \mathcal{P}, \;\; \Pi_{ij}(m) = \sum_{k \in E} \Pi_{ik} \Pi_{kj}(m-1) \quad \text{for} \quad m \in \mathbb{N}$$

Examples

Example (positive)

A transition matrix of some irreducible and aperiodic finite Markov chain:

$$\Pi = \begin{bmatrix} 0 & 0 & 0.1 & 0.1 & 0.7 & 0.1 \\ 0.2 & 0.3 & 0.4 & 0.1 & 0 & 0 \\ 0.1 & 0.9 & 0 & 0 & 0 & 0 \\ 0.2 & 0.2 & 0.3 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.4 & 0.3 & 0.1 & 0.2 \end{bmatrix}$$

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The corresponding unitary matrix (with the same pattern of zero/ non-zero elements):

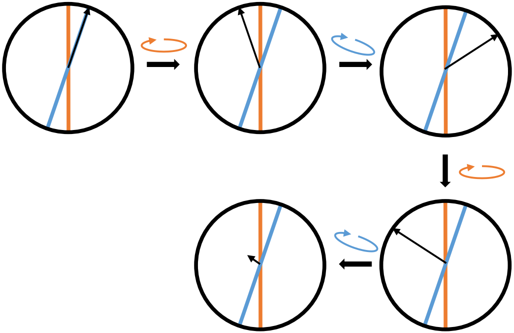
$$U = \begin{bmatrix} 0 & 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}.$$

Bounding the Number of Strokes

Bounding the Number of Strokes – INTUITIONS

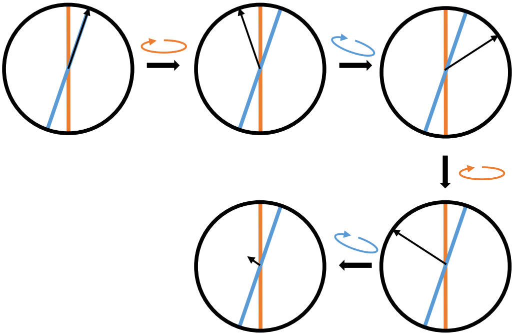
A single-qubit system with:

- F_1, F_2 – states diagonal in the σ_z and $\cos\alpha\sigma_z + \sin\alpha\sigma_x$ eigenbases
- $\mathcal{F}_1$, $\mathcal{F}_2$ – unitary rotations around the appropriate axes



Bounding the Number of Strokes – INTUITIONS

- $\alpha = \pi/2$: just one operation is needed.
- $\alpha < \pi/2$: $\lceil \pi/(2\alpha) \rceil$ operations (done subsequently by Alice and Bob) are needed.



Bounding the Number of Strokes Needed to Generate All Operations (QUBIT CASE)

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Theorem (qubit case)

Alice and Bob,
restricted to apply operations from $\mathcal{F}_A$ and $\mathcal{F}_B$, consisting of rotations about three-dimensional real unit vectors $\hat{m}$ and $\hat{n}$ (rotated with respect to each other by α),
can generate any unitary matrix with the minimal number of alternated rotations about vectors $\hat{m}$ and $\hat{n}$ equal to

$$\left\lceil \frac{\pi}{\alpha} \right\rceil + 1.$$

Bounding the Number of Strokes Needed to Generate All Operations – THE LOWER BOUND

Theorem

The number $2N$ of operations that Alice and Bob need to perform (N by Alice, and N Bob) in order to generate an arbitrary unitary matrix of order n is bounded from below by

$$\frac{\log(n-1)}{\log((n-2)c_U+1)} \quad \text{for} \quad n \in \mathbb{N}\backslash\{1,2\} \qquad \text{and} \qquad \frac{1}{c_U} \quad \text{for} \quad n = 2,$$

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where

$$c_U := \max_{a,b\in\{1,\ldots,n\}:\ a\neq b} \sum_{j=1}^{n} |u_{j,a}|\,|u_{j,b}|$$

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(equivalently, $c_U = \max_{a,b\in\{1,\ldots,n\}:\ a\neq b} \langle a|(X_U^T X_U)|b\rangle$ with $x_{i,j} = |u_{i,j}|$ for all $i,j \in \{1,\ldots,n\}$).

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Every circulant matrix can be written as a ***product of a Fourier transform, a diagonal matrix, and an inverse Fourier transform****.*

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$n + (n-1)(2\mathbf{M}+1)$ factors

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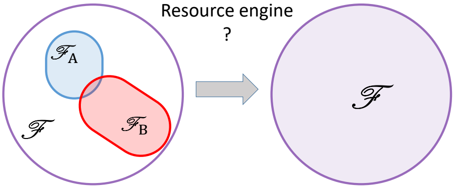
M – number of operations needed to generate a Fourier matrix **(open)**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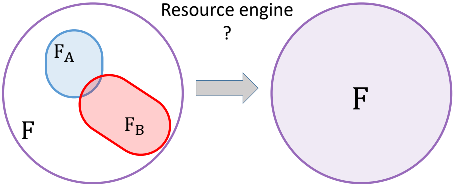
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For any two bases $\{|a_i\rangle\}_{i=1}^n$ and $\{|b_i\rangle\}_{i=1}^n$ of an *n*-dimensional Hilbert space there exist at least 2^{n-1} states $|\psi_*\rangle$ unbiased in both these bases

$$|\langle a_i|\psi_*\rangle| = |\langle b_i|\psi_*\rangle| = \frac{1}{\sqrt{n}} \quad \text{for all} \quad i \in \{1, \ldots, n\}.$$

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[M. Idel, M.M. Wolf, **Linear Algebra Its Appl.** 471, 76–84 (**2015**)]
[K. Korzekwa, D. Jennings, and T. Rudolph, **Phys. Rev. A** 89, 052108 (**2014**)]

Condition on Getting the Optimal State – Within a Single Stroke (QUBIT CASE)

Theorem

Let $U \in \mathcal{U}_2(\mathbb{C})$ be such that

$$U = e^{i\phi} \begin{bmatrix} e^{i\varphi_0}\cos(\varphi) & -e^{-i\varphi_1}\sin(\varphi) \\ e^{i\varphi_1}\sin(\varphi) & e^{-i\varphi_0}\cos(\varphi) \end{bmatrix}$$

with $\varphi \in [\pi/8, 3\pi/8]$.

THEN Alice and Bob can produce a mutually coherent state after performing only two operations

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For bigger n it follows form generalized polygon inequalities.

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Let $U \in \mathcal{U}_n(\mathbb{C})$. The necessary condition for the existance of $D \in \mathcal{DU}_n(\mathbb{C})$ such that $U^\dagger DU$ has a flat column:

$$\exists_{l\in\{1,\dots,n\}} \forall_{m\in\{1,\dots,n\}} \max_{i\in\{1,\dots,n\}} |u_{m,i}\bar{u}_{l,i}| \leqslant \frac{1}{2}\left(\sum_{j=1}^{n} |u_{m,j}\bar{u}_{l,j}| + \frac{1}{\sqrt{n}}\right).$$

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Corollary

IF there exist: a permutation matrix Π and $D \in \mathcal{DC}_n(\mathbb{C}$ such that

$$\|U - D\Pi\|_{HS}^2 < 2 - 2\left(1/2\left(1 + n^{-1/2}\right)\right)^{1/2},$$

THEN Alice and Bob are NOT ABLE to generate a mutually coherent state after performing only two operations.

Fixed Point Constraints (Thermodynamics)

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Notation:

- $\gamma = (\gamma_1, \ldots, \gamma_n)$, $\Gamma = (\Gamma_1, \ldots, \Gamma_n)$ – arbitrary thermal states (probability vectors) with respect to different temperatures.
- $F_A = \{\gamma\}$, $F_B = \{\Gamma\}$.
- $\mathcal{F}_A$, $\mathcal{F}_B$ – the sets of all these stochastic operations for which γ and Γ are the fixed points.

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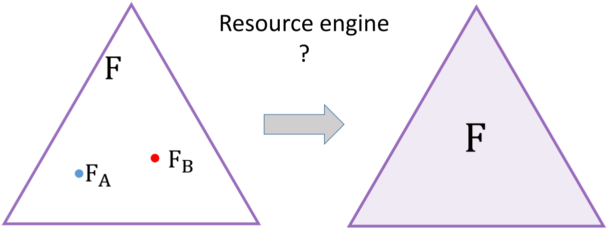
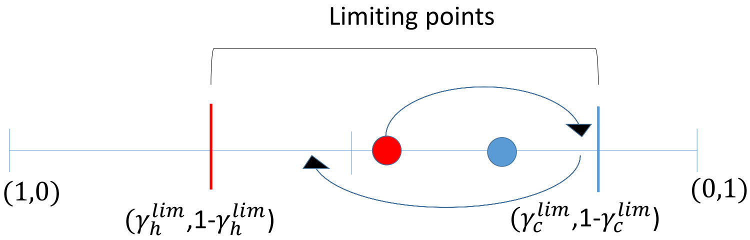
Aim: producing an arbitrary state from an n-dimensional simplex.



Condition on Getting All States (BIT CASE)

$$\Gamma = (\gamma_h, 1 - \gamma_h) \qquad \gamma = (\gamma_c, 1 - \gamma_c)$$



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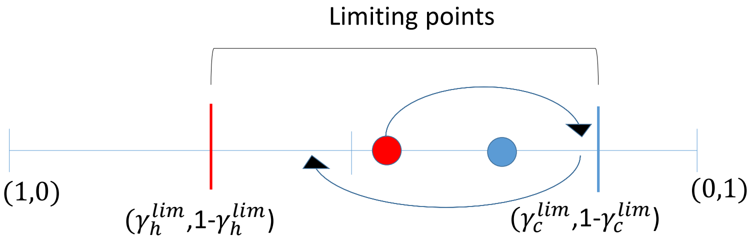

$$\gamma_h^{max} = \frac{(2\gamma_h - 1)\gamma_c}{\gamma_h + \gamma_c - 1} \qquad \gamma_c^{max} = \frac{(2\gamma_c - 1)\gamma_h}{\gamma_h + \gamma_c - 1}$$

Condition on Getting All States – While Having Access to a Maximally Mixed State

Theorem

IF $\Gamma = (1/n, \ldots, 1/n)$ *and* $\gamma \neq \Gamma$,
THEN Alice and Bob can produce any state of an n-dimensional simplex (and the rate of convergence is exponential).

Condition on Getting All States – While Having Access to a Maximally Mixed State

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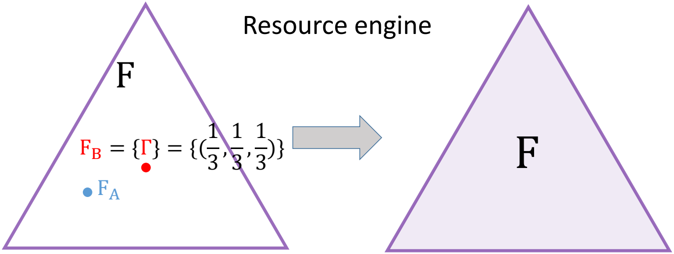
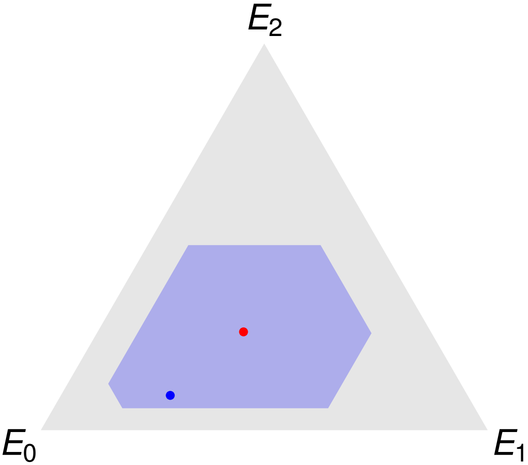
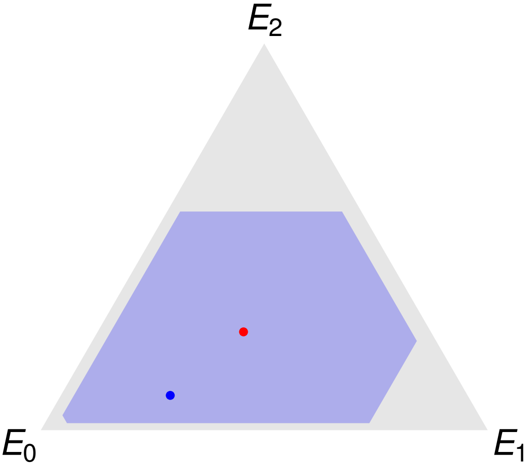


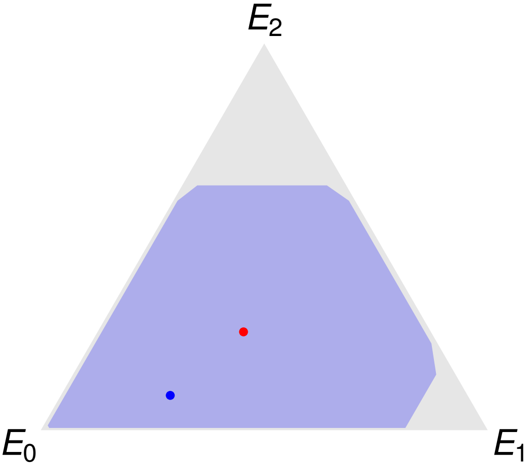
States Achievable After *n* Strokes



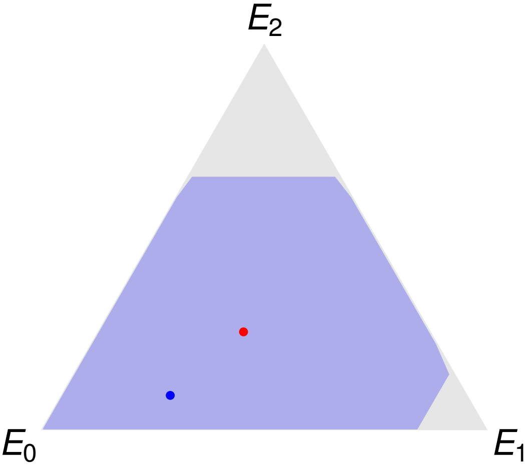
States Achievable After n Strokes



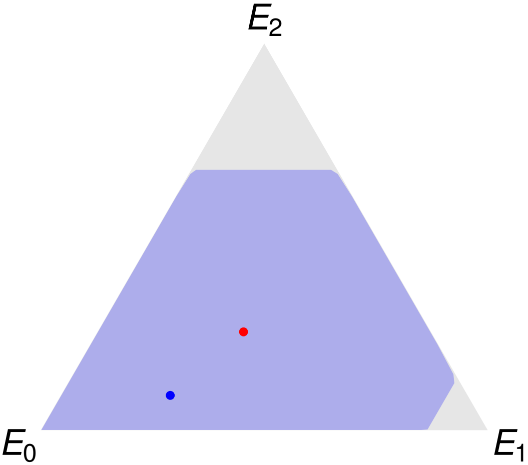
States Achievable After n Strokes



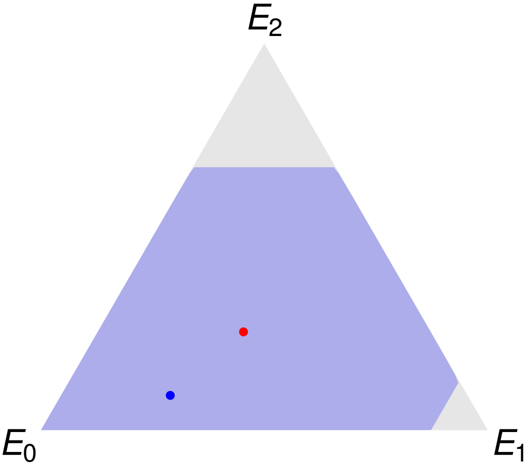
States Achievable After *n* Strokes



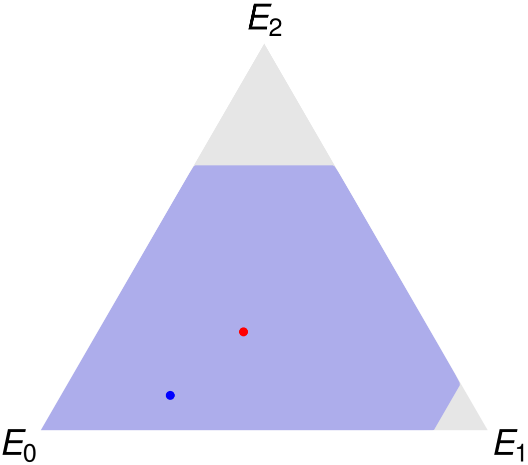
States Achievable After n Strokes



States Achievable After n Strokes



States Achievable After n Strokes



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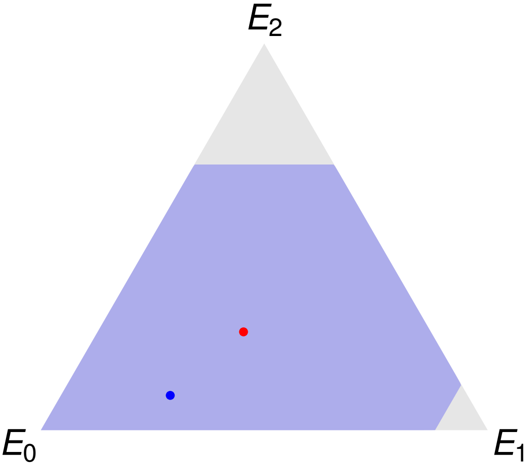

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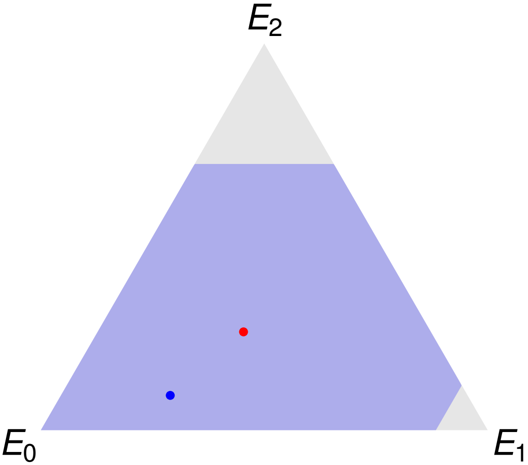
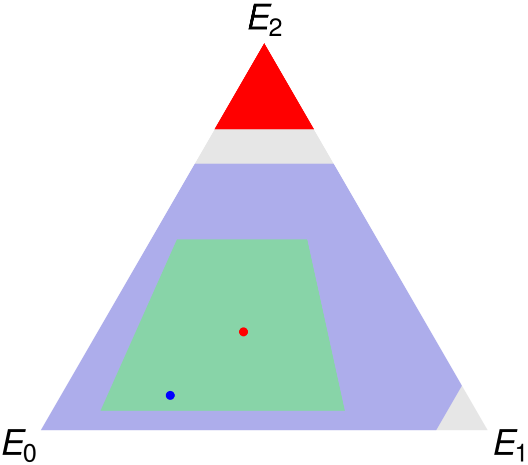
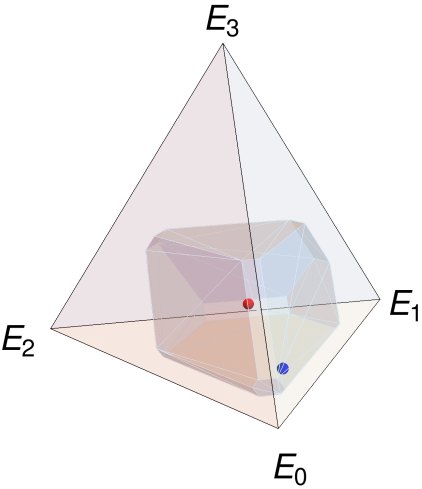


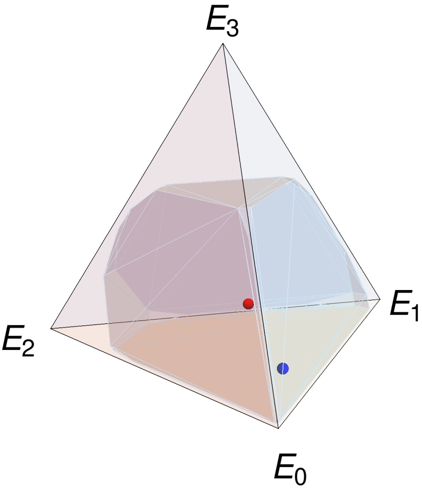
States Achievable After n Strokes



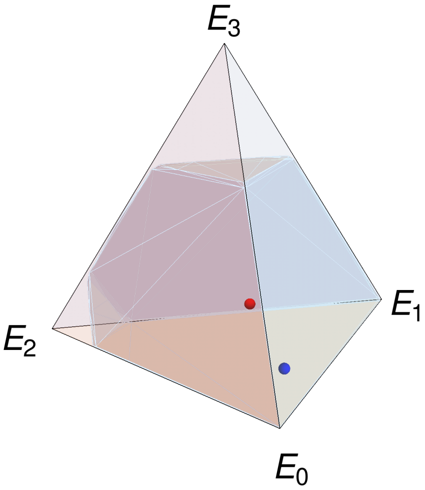
States Achievable After n Strokes



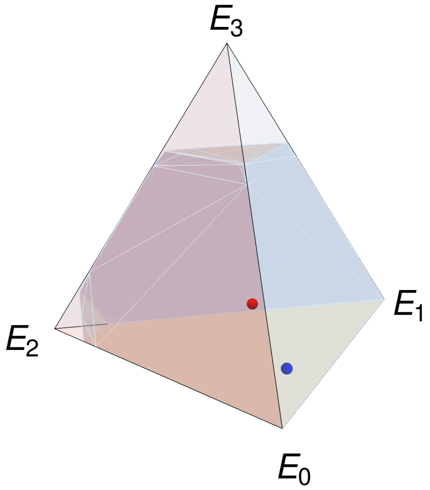
States Achievable After n Strokes



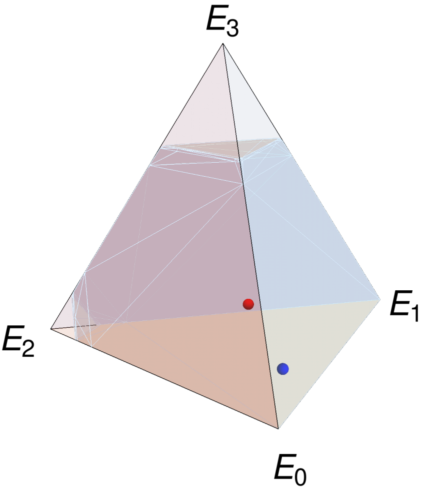
States Achievable After *n* Strokes



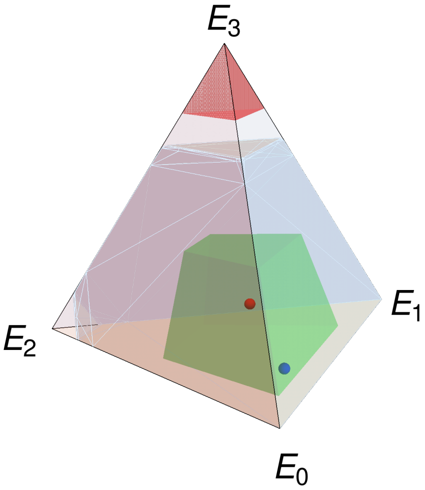
States Achievable After n Strokes



States Achievable After n Strokes



States Achievable After n Strokes



Acknowledgments

Thank You for Your Attention

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