Universal trade-off structure between symmetry, irreversibility and quantum coherence for quantum processes

Hiroyasu Tajima
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HT and K. Saito arXiv:2103.01876 (2021)

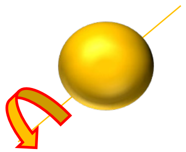
HT, R. Takagi and Y. Kuramochi arXiv:2206.11086 (2022)

Accepted by QIP 2023 talk!

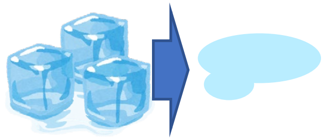
Topic of today's talk

A universal trade-off between

Symmetry



Irreversibility

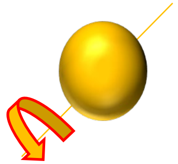


Quantum coherence

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

Background

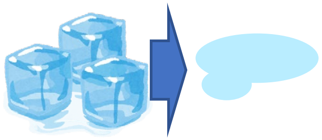
Symmetry



Guiding principle in modern physics

High energy physics, condensed matter, and quantum information (QI)

Irreversibility



Central concept in thermodynamics and stochastic mechanics

Also related to quantum error corrections

Quantum coherence

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

Unique property of quantum mechanics

Origin of quantum advantage in many tasks

Main result: a universal trade-off between these three.

Brief introduction of Main result

Main result :

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}}+\Delta} \leq \delta \text{ or } \sqrt{\delta}$$

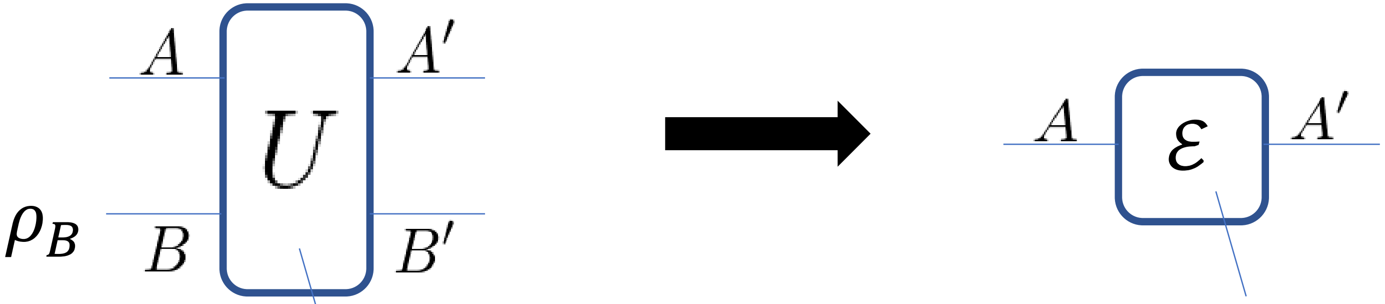
H. Tajima and K. Saito, arXiv:2103.01876 (2021)
H. Tajima, R. Takagi, Y. Kumorachi, arXiv:2206.11086 (2022).

It has many applications:

Ex : Quantum computation, error corrections, measurement theory,
Quantum thermodynamics, and black hole physics

Let’s see the details.

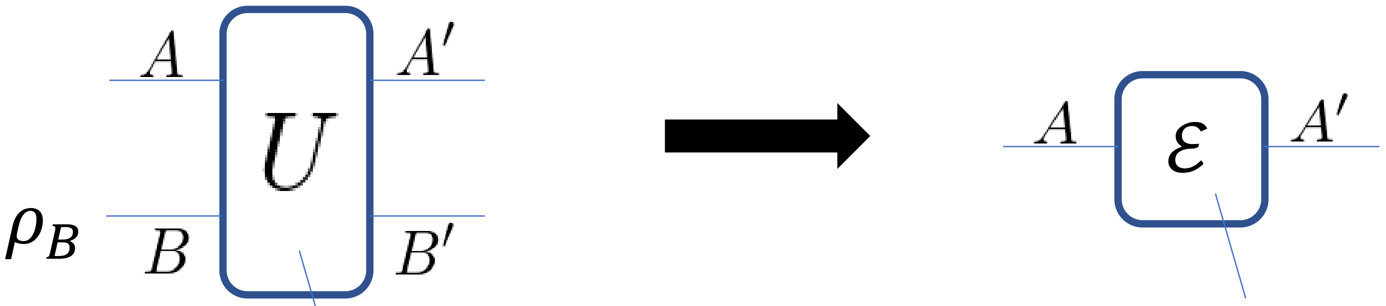
Setup



$$X_{A'} + \mathrm{X}_{B'} = U^\dagger (X_A + X_B) U$$

$$\mathcal{E}(\ldots) := \mathrm{Tr}_{B'}[U(\ldots \otimes \rho_B) U^\dagger]$$

Setup



$$X_{A'} + X_{B'} = U^\dagger (X_A + X_B) U$$

$$\mathcal{E}(\dots) := \mathrm{Tr}_{B'}[U(\dots \otimes \rho_B)U^\dagger]$$

Result

Change of local conserved charge X_A by $\mathcal{E}$

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}} + \Delta} \leq \delta \text{ or } \sqrt{\delta}$$

Irreversibility of $\mathcal{E}$

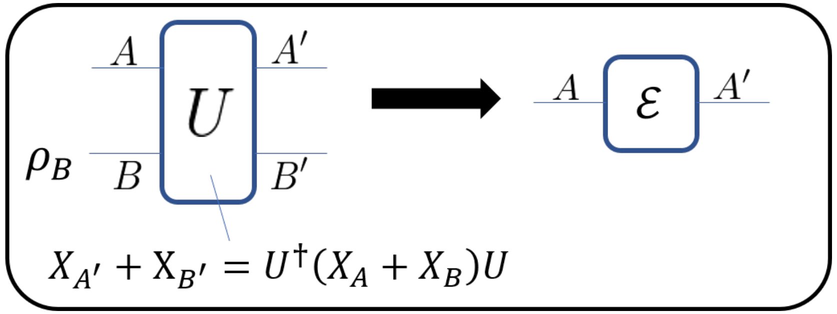
Positive constant

Quatnum coherence of X_B in B
=Asymmetry
=Fisher information

Because of the time constraint, I only explain important properties of these quantities

Properties of key quantities

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}}+\Delta} \leq \delta \text{ or } \sqrt{\delta}$$



Irreversibility δ : function of $\mathcal{E}$ and a “test ensemble” $\{\mathrm{p_k}, \rho_k\}$

Properties of key quantities

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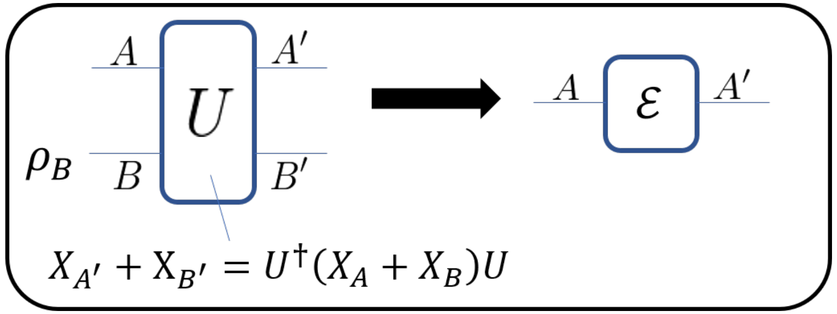


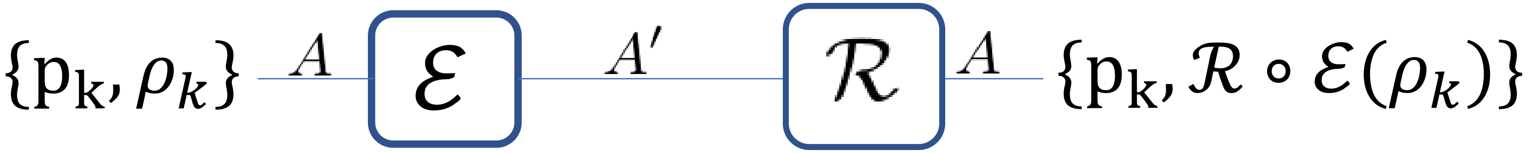
Irreversibility δ : function of $\mathcal{E}$ and a "test ensemble" $\{\mathrm{p_k}, \rho_k\}$

$$\delta := \min_{\mathcal{R}:A' \to A} \sqrt{\sum_k p_k \delta_k^2}$$

$$\delta_k := D_F(\rho_k, \mathcal{R} \circ \mathcal{E}(\rho_k))$$

recovery error for ρ_k

$$D_F\,(\rho,\,\sigma) := \sqrt{1 - F^2(\rho, \sigma)}$$



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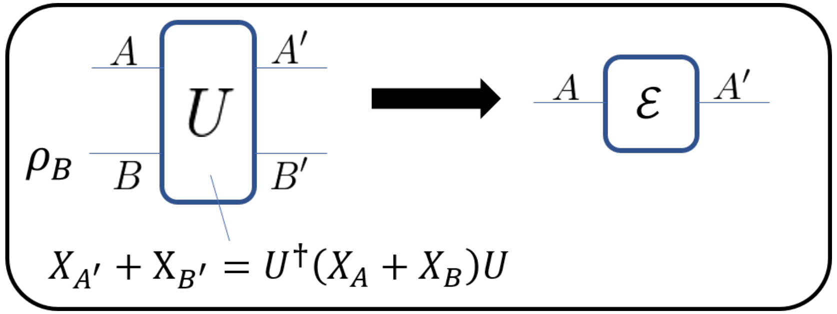

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$$\delta := \min_{\mathcal{R}:A' \to A} \sqrt{\sum_k p_k \delta_k^2}$$

Property: δ gives lower bounds for various irreversibility measures.

$$\delta \leq \sqrt{\Sigma}, \quad \delta \leq \delta_Q, \quad \text{and } \delta \leq \frac{\delta_P}{\sqrt{2}},$$

e.g., it bounds the entropy production, the entanglement-fidelity recovery error, and the error of Petz map recovery, etc.

Hiroyasu Tajima, 2103.01876 & 2206.11086

Properties of key quantities

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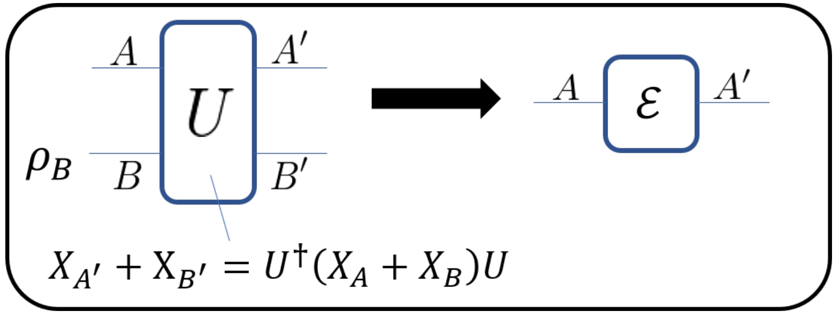

$\boldsymbol{\mathcal{F}}$ **:** Quantum coherence of X_B in ρ_B measured by QFI

$$\boxed{\mathcal{F} := \mathcal{F}_{\rho_B}(X_B)}$$

SLD-Quantum Fisher information

$$\mathcal{F}_\rho(X) := \lim_{t\to 0} \frac{D_F(\rho, e^{-iXt}\rho e^{iXt})^2}{t^2}$$

Property: SLD-Quantum Fisher information is a standard measure of coherence in the sense of the resource theory of asymmetry.

C. Zhang, et al., Physical Review A 96, 042327 (2017).
R. Takagi, Scientific Reports 9, 14562 (2019).
I. Marvian, Nature Communications 11, 25 (2020).

Properties of key quantities

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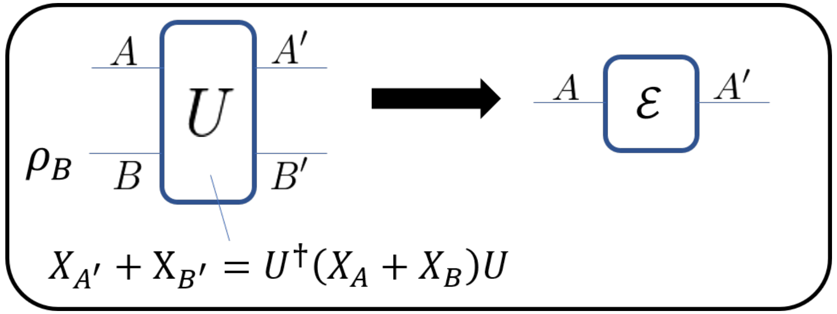


$\mathcal{C}$ **:** Degree of change of local charge

$$\boxed{\mathcal{C} := \sqrt{\sum_{k\neq k'} p_k p_{k'} \mathrm{Tr}[(\rho_k - \rho_{k'})_+ Y_A (\rho_k - \rho_{k'})_- Y_A]}}$$

$(O)_{\pm}$:=Positive (negative) part of the Hermitian operator O

$Y_A := X_A - \mathcal{E}^{\dagger}(X_{A'})$ **Operator of change of local charge**

Property: ・When the test states are orthogonal pure states $\{|\psi_k\rangle\}$,

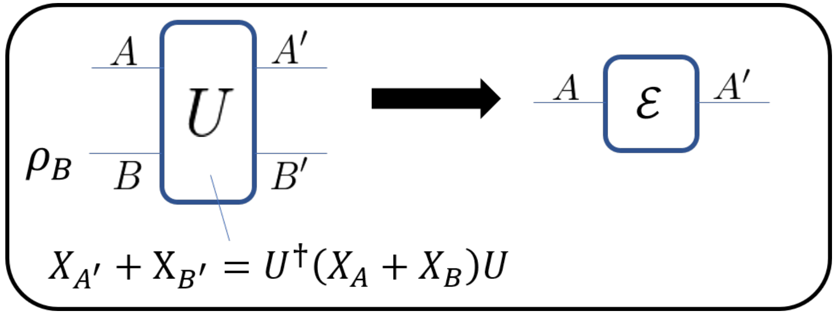
$$\mathcal{C} = \sum_{k\neq k'} p_k p_{k'} |\langle\psi_k|Y_A|\psi_{k'}\rangle|^2$$

Sum of absolute values non-diagonal parts

・$\mathcal{E}$ changes X_A non-trivially (i.e.$Y_A \not\propto 1_A$)$\Rightarrow$ $\mathcal{C} > 0$

Main result

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}}+\Delta} \le \delta \text{ or } \sqrt{\delta}$$

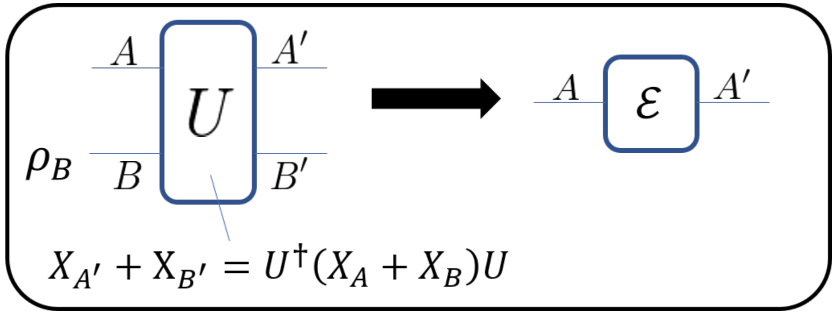


For an arbitrary test ensemble $\{p_k, \rho_k\}$

For the test states $\{\rho_k\}$ are orthogonal each other,
i.e., $\mathrm{F}(\rho_k, \rho_{k'}) = 0$ for $\mathrm{k} \neq k'$,

Messages of main result

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}}+\Delta} \leq \delta \text{ or } \sqrt{\delta}$$



1. $\mathcal{C}>0 \Rightarrow \delta>0$

➡ When $\mathcal{E}$ changes the local charge nontrivially, $\mathcal{E}$ must be irreversible.

2. The coherence in B $(= \mathcal{F})$ can mitigate the irreversibility.

Take home message:

Under global *symmetry*, local charge cannot be changed without *irreversibility*.
But the irreversibility can be mitigated by *quantum coherence*.

➡S-I-Q trade-off!

Applications of S-I-Q tradeoff

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}}+\Delta} \leq \delta \ or \ \sqrt{\delta}$$

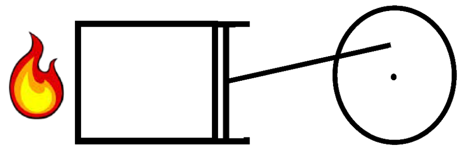
- Restriction on Petz map recovery
  - General trade-off between entropy production and coherence
  - Lower bound of coherence cost for arbitrary quantum channel in RToA
- Application to black holes 2: Universal error bound for m-bits classical information recovery.
- Wigner-Araki-Yanase type theorems on measurements
- General restriction on Quantum information recovery
  - Extensions of Eastin-Knill theorems on covariant codes
  - Application to black holes 1: Limitation on quantum information recovery from BH
  - Wigner-Araki-Yanase type theorems on unitary gates
- No-go theorem on Hamiltonian quench
- ⋮

Applications of S-I-Q tradeoff (simple version)

Main Theorem:
Trade-off between symmetry, irreversibility and coherence

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}}+\Delta} \leq \delta \text{ or } \sqrt{\delta}$$

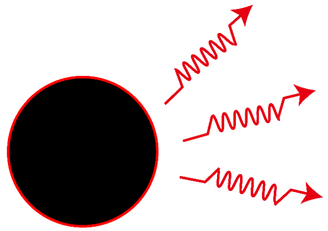
Thermodynamic processes:



Trade-off between entropy production and quantum coherence:

$$\sqrt{\Sigma_\beta(\rho)} \geq \frac{\mathcal{C}^2}{(\sqrt{\mathcal{F}_{\rho_C}(X_C)}+\Delta)^2}$$

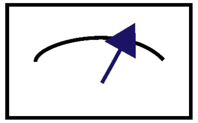
black hole physics:



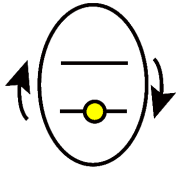
Universal error bound for m-bits classical information recovery:

$$\delta_H \geq \frac{m}{4} \times \frac{1}{\left(1+\frac{3}{a\gamma}\right)^2}$$

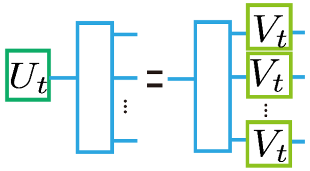
Quantum information processings:



Quantitative WAY theorem for fidelity error



Coherence costs on gate implemetations



Extension of Eastin-Knill theorem

⋮

Applications of SIQ tradeoff (simple version)

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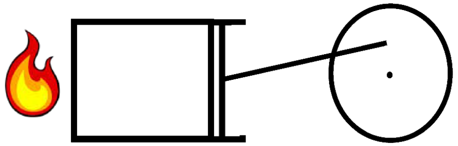

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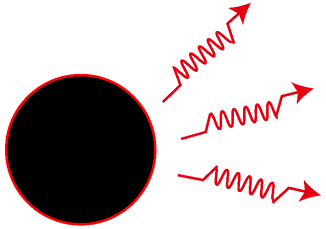

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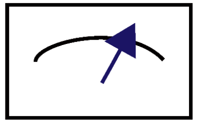

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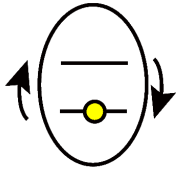

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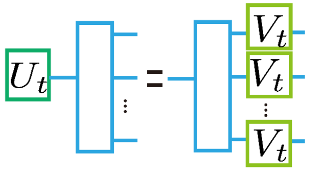

Extension of Eastin-Knill theorem

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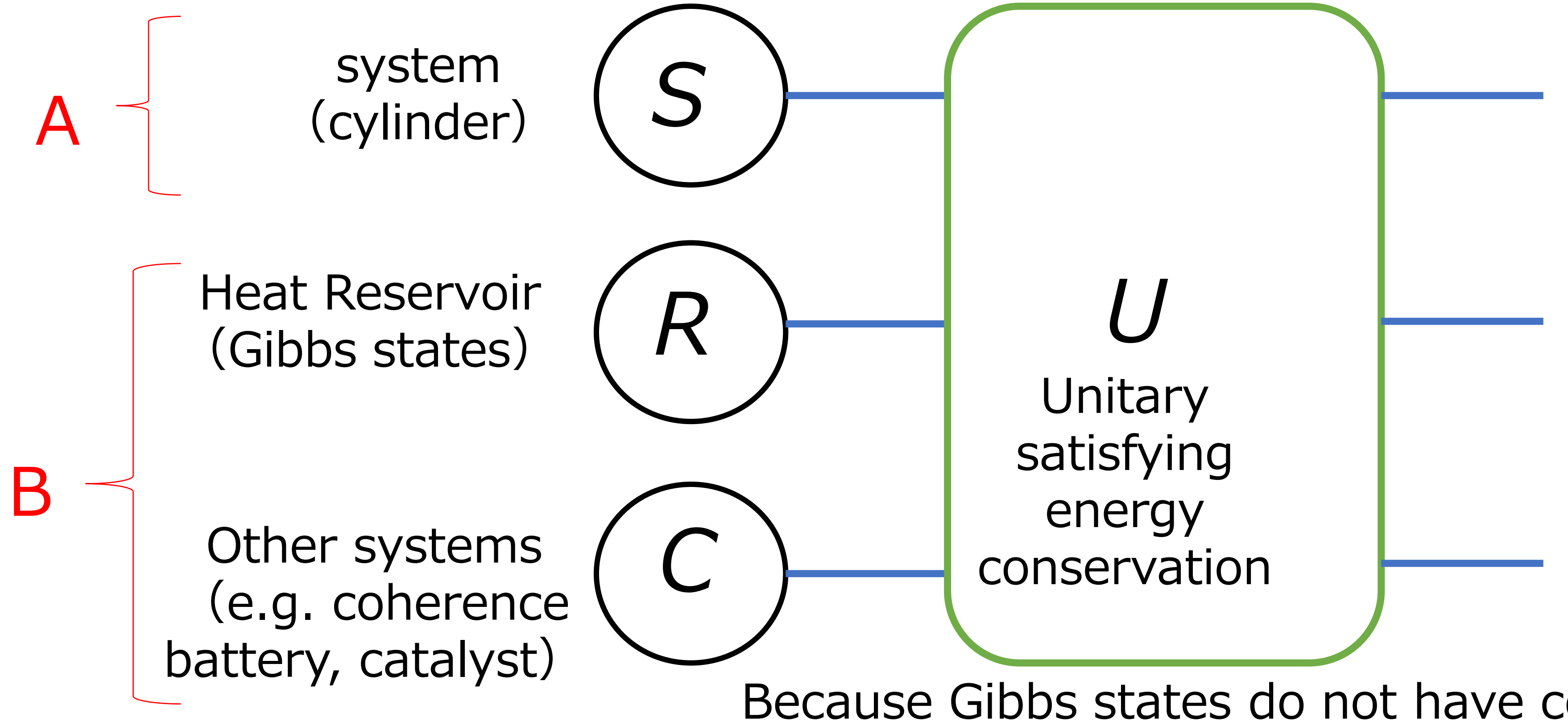
Application to thermodynamic processes

Let us see how our theorem applies to thermal processes.

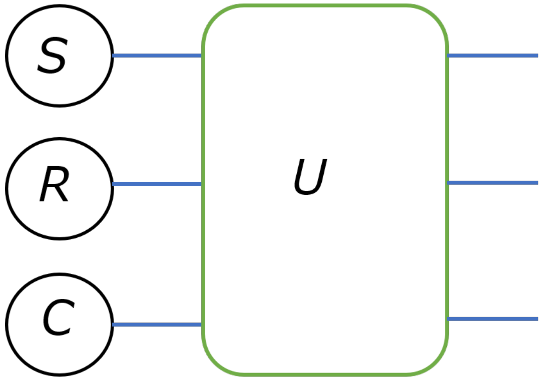
For simplicity, let us consider the single-bath case. In this case, we can formulate a thermal process as follows:



Because Gibbs states do not have coherence of energy, we can evaluate the amount of coherence in C by our theorem

Application to thermodynamic processes

Under this formulation, we can evaluate the coherence cost for implementing an arbitrary channel $\mathcal{E}$.



$$\mathcal{F}_{\mathrm{cost}} \geq \frac{\mathcal{C}^2}{\sqrt{\Sigma}} - \Delta^2$$

This inequality shows a trade-off relation between the generalized entropy production Σ of $\mathcal{E}$ and coherence cost $\mathcal{F}_{\mathrm{cost}}$ of $\mathcal{E}$.

Trade-off between thermodynamic irreversibility and quantum coherence under global symmetry!

Applications of S-I-Q tradeoff (simple version)

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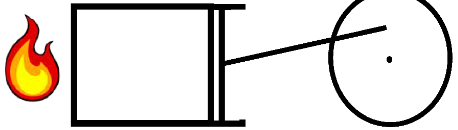

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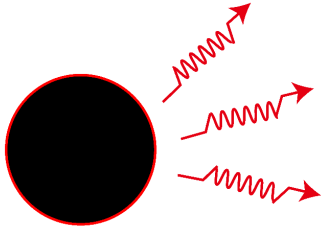

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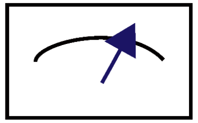

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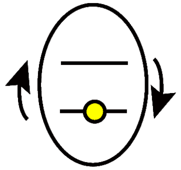

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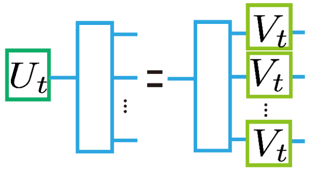


Extension of Eastin-Knill theorem

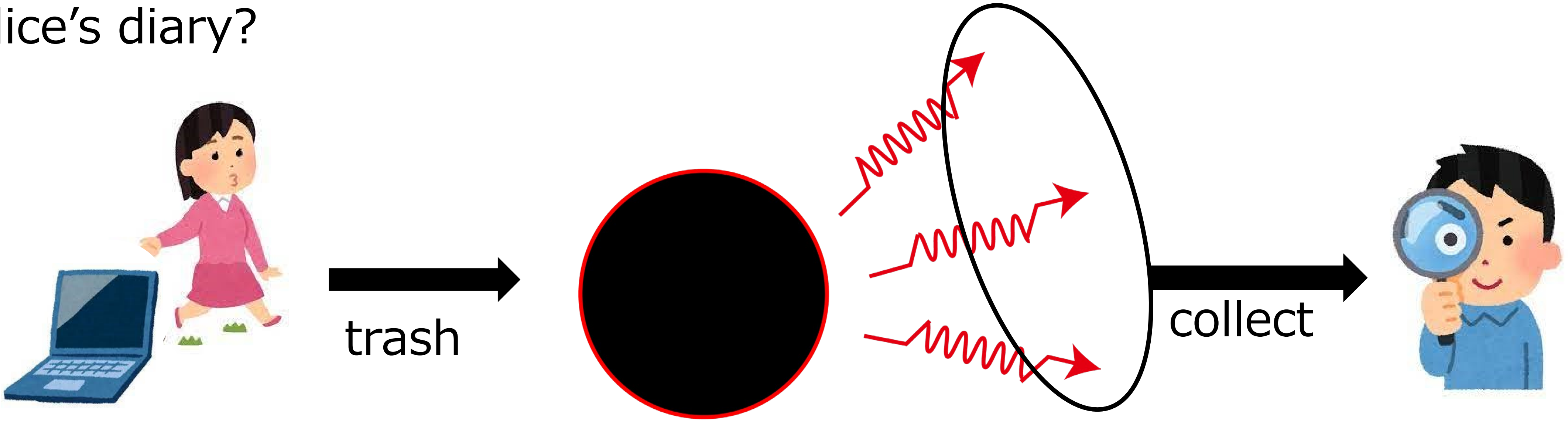
⋮

Application to black hole physics

Hayden-Preskill (HP) thought experiment

- Alice trashes her computer (“diary” in the original paper) to a black hole.
- Bob tries to recover the data in her diary from the Hawking radiation.
  (Bob have collected *all* radiation since the black hole emerged.)

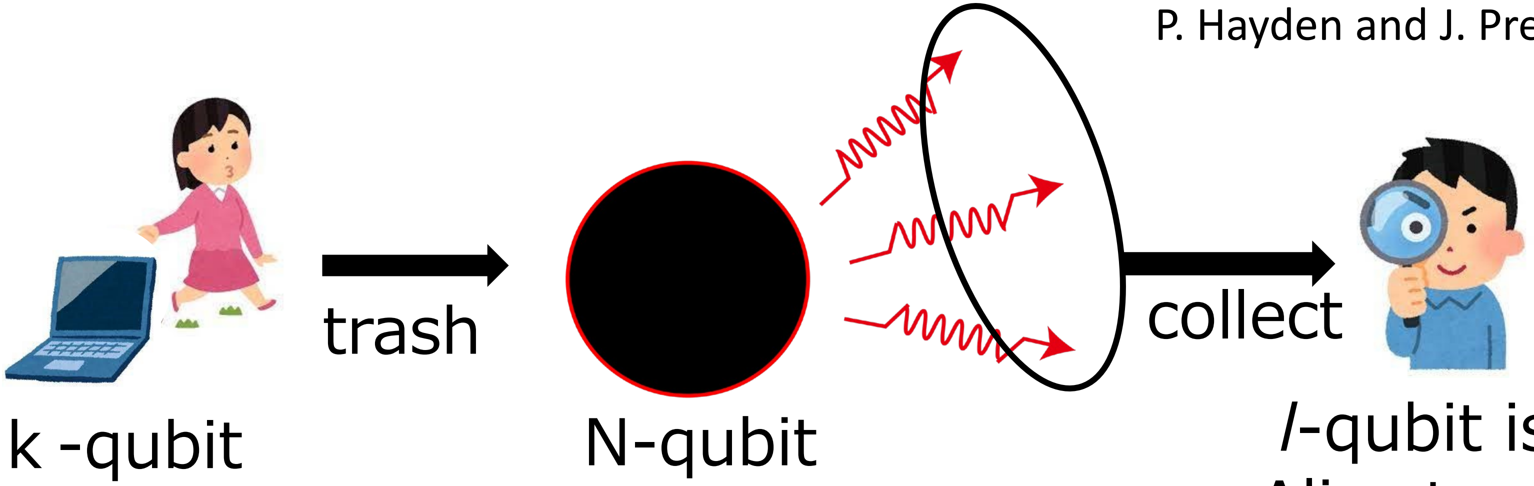
How much radiation does Bob need to recover the original information in Alice’s diary?



Our result is applicable to this situation.

Hayden and Preskill's analysis

P. Hayden and J. Preskill, JHEP 0709, 120 (2007).



l-qubit is collected after Alice trashes her diary.

Assumptions:

- The black hole, the computer and the radiation consist of qubits.
- The dynamics of the black hole is a typical Haar random unitary.

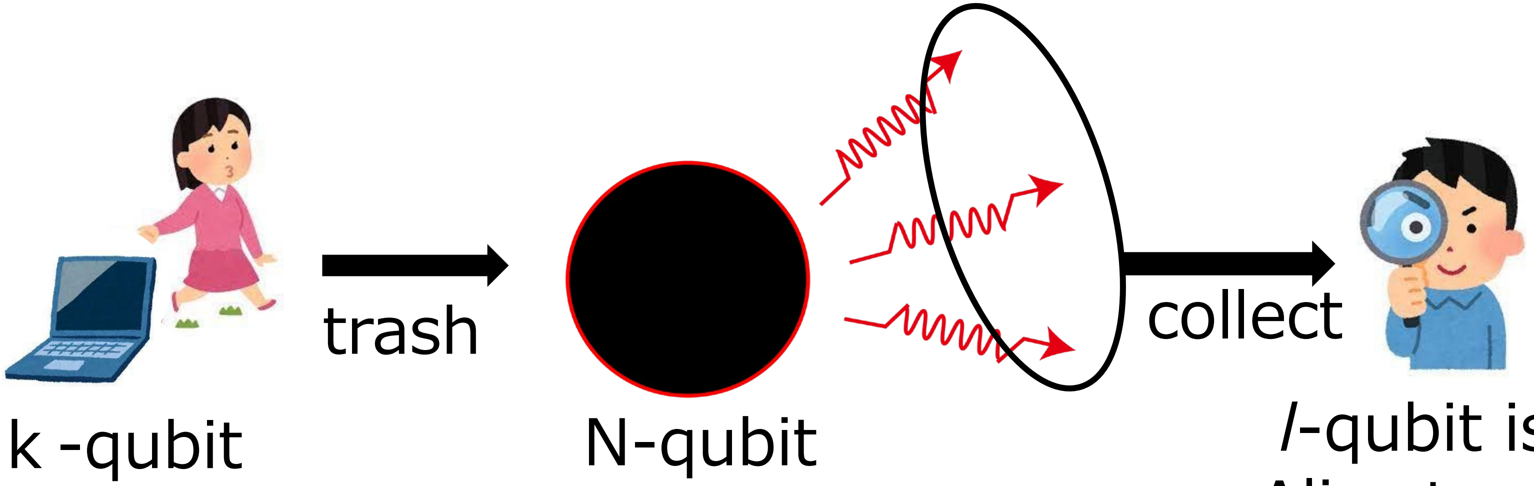
Result:

$$\delta_Q \leq \text{const.} \times 2^{-(l-k)}$$

When l is little bit larger than k, Bob can recover the information in Alice's diary almost perfectly.

➡Black holes are informative mirrors!

Effect of energy conservation on HP thought experiment



l-qubit is collected after Alice trashes her diary.

Hayden and Preskill's analysis, however, does not assume energy conservation. What happens when energy conservation is assumed?

Several pioneering works suggest that the information recovery gets late in that cases.

B. Yoshida, Phys. Rev. D **100**, 086001 (2019).
J. Liu, Phys. Rev. Research **2**, 043164 (2020).

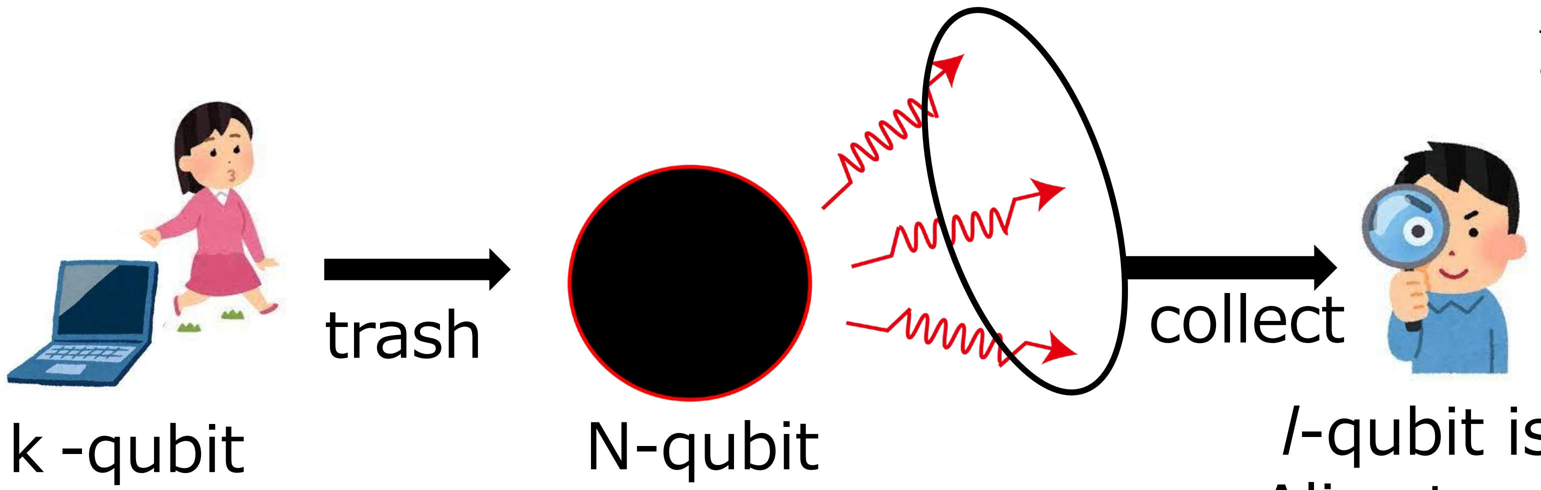
In particular, a lower bound of recovery error is given. (In this article, upper bounds are also discussed.) Y. Nakata, E. Wakakuwa, M. Koashi, arXiv:2007.00895 (2020)

But the previous lower bound needs extra assumptions and approximations.

Our theorem gives a rigorous lower bound of error without any extra assumptions and approximations.

Effect of energy conservation on HP thought experiment

H. Tajima and K. Saito, arXiv:2103.01876 (2021).



l-qubit is collected after Alice trashes her diary.

Assumptions:

・The black hole, the computer and the radiation consist of qubits.
・The dynamics of the black hole is a typical Haar random unitary satisfying the energy conservation.

Result:

$$\frac{\text{const.}}{1+\frac{2\sqrt{2}}{\sqrt{\gamma}}} \le \delta_Q$$

$$\gamma = 1 - \frac{l}{N+k}$$

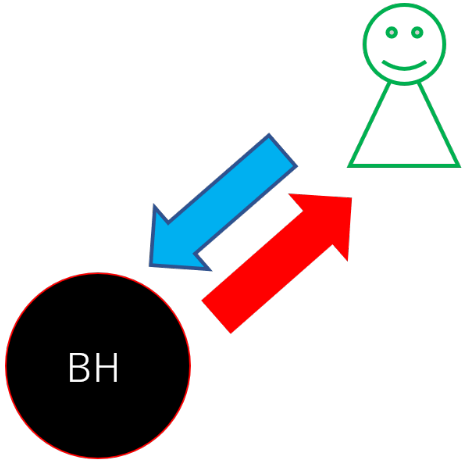
= the remaining ratio of the black hole

The information recovery has inevitable error depending only on the remaining ratio of the black hole.

Effect of energy conservation on HP thought experiment

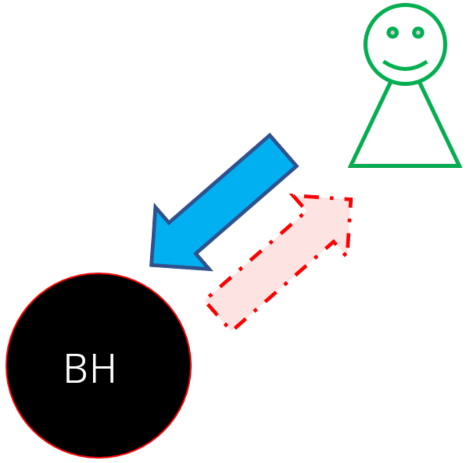
Our result rigorously show that energy conservation changes the speed of the information escape from black holes radically:

Case without symmetry

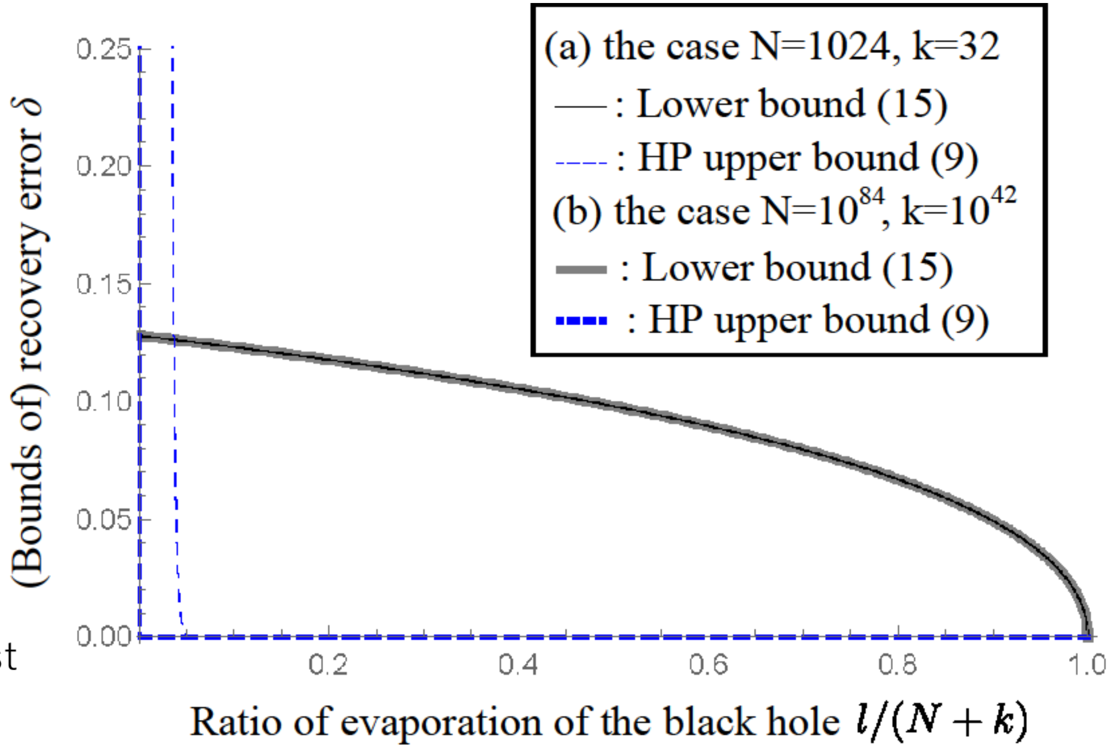


Black hole reflects all of the thrown quantum information.

Case with symmetry



Black hole reflects only a part of the thrown quantum information. The rest cannot escape from the black hole.



$$\delta_Q \le \text{const.}\times 2^{-(l-k)}$$

$$\frac{\text{const.}}{1+\frac{2\sqrt{2}}{\sqrt{\gamma}}} \le \delta_Q$$

Applications of S-I-Q tradeoff (simple version)

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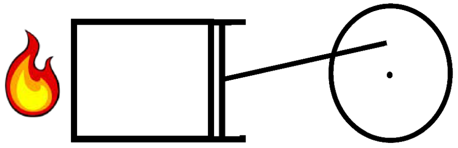
Thermodynamic processes:



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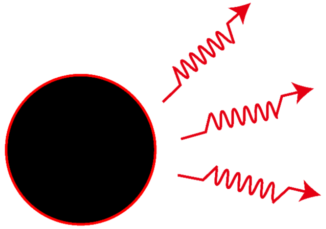

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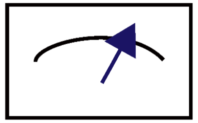
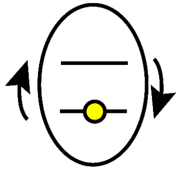
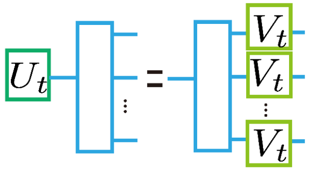
Quantum information processings:



Quantitative WAY theorem for fidelity error



Coherence costs on gate implemetations



Extension of Eastin-Knill theorem

⋮

Application to quantum information processing

There are various restrictions on quantum information processing imposed by symmetry:

Measurement : WAY theorem

Inversely proportional relation between

Measurement error ⇔ Coherence cost

E. P. Wigner, Z. Phys. 133, 101 (1952).
H. Araki and M. M. Yanase, Phys. Rev.120, 622 (1960).
M. M. Yanase, Phys. Rev. 123, 666 (1961).

M. Ozawa, Phys. Rev. Lett. 88, 050402 (2002).
K. Korzekwa, M.Res. thesis (2013).
H. Tajima and H. Nagaoka, arXiv:1909.02904 (2019)

Unitary gates : unitary WAY theorem

Inversely proportional relation between

Implementation error ⇔ Coherence cost

M. Ozawa, Phys. Rev. Lett. 89, 057902 (2002).
H. Tajima, N. Shiraishi and K. Saito, Phys. Rev. Lett. 121, 110403 (2018).
H. Tajima, N. Shiraishi and K. Saito,Phys. Rev. Research. 2, 043374 (2020).

Covariant codes : Eastin-Knill theorem

Inversely proportional relation between

Recovery error ⇔ Qubits required by the code

B. Eastin and E. Knill, Phys. Rev. Lett. 102, 110502 (2009).
P. Faist, S. Nezami, V. V. Albert, G. Salton, F. Pastawski, P. Hayden, and J. Preskill, Phys. Rev. X 10, 041018 (2020).

SIQ tradeoff behaves a unified theorem of these restrictions.

It recovers and extends these results as corollaries.

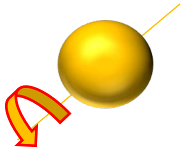
Summary

HT and K. Saito
arXiv:2103.01876 (2021)
HT, R. Takagi and Y. Kuramochi
arXiv:2206.11086 (2022)

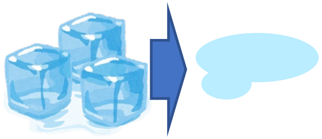
We find a universal trade-off relation between symmetry, irreversibility and quantum coherence:

$$\boxed{\frac{\mathcal{C}}{\sqrt{\mathcal{F}}+\Delta} \leq \delta \text{ or } \sqrt{\delta}}$$

Symmetry



Irreversibility



Quantum coherence

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

It has various applications, e.g., thermodynamic processes, measurements, error correcting codes, computation gates, and black hole physics.

Appendix: Other applications to BH physics

HT, R. Takagi and Y. Kuramochi
arXiv:2206.11086 (2022)

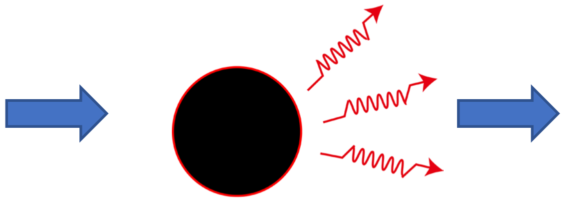
SIQ tradeoff can give other applications about BH physics.

For example, we can evaluate when we throw m bits of *classical information* into a BH, how many bits will be unrecoverable on average:

$$\delta_H \geq \frac{m}{4}\left(1+\frac{3}{a\gamma}\right)^2$$

Classical m bits

011100011010···0011101101



About m/4 bits will be unrecoverable until the black hole almost evaporates

0✖11✖001✖010···✖0✖11011✖1