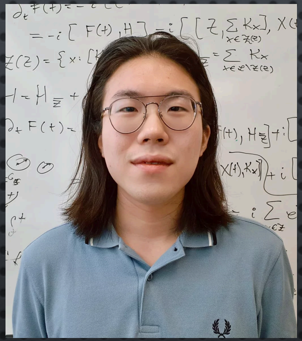
CATALYSIS IN ACTION
VIA ELEMENTARY THERMAL OPERATIONS

Jeongrak Son



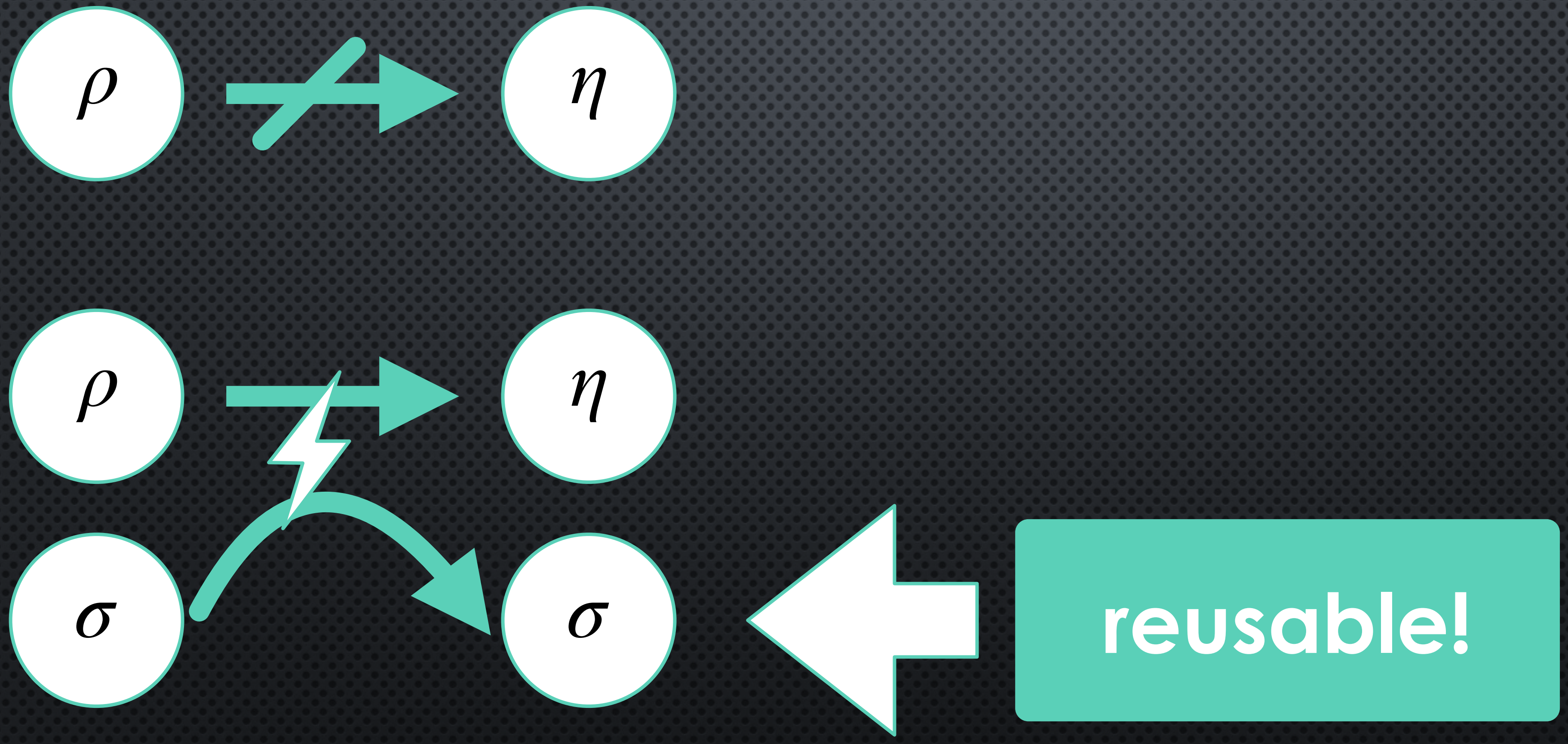
w/ Nelly H. Y. Ng

arXiv:2209.15213

im.Q

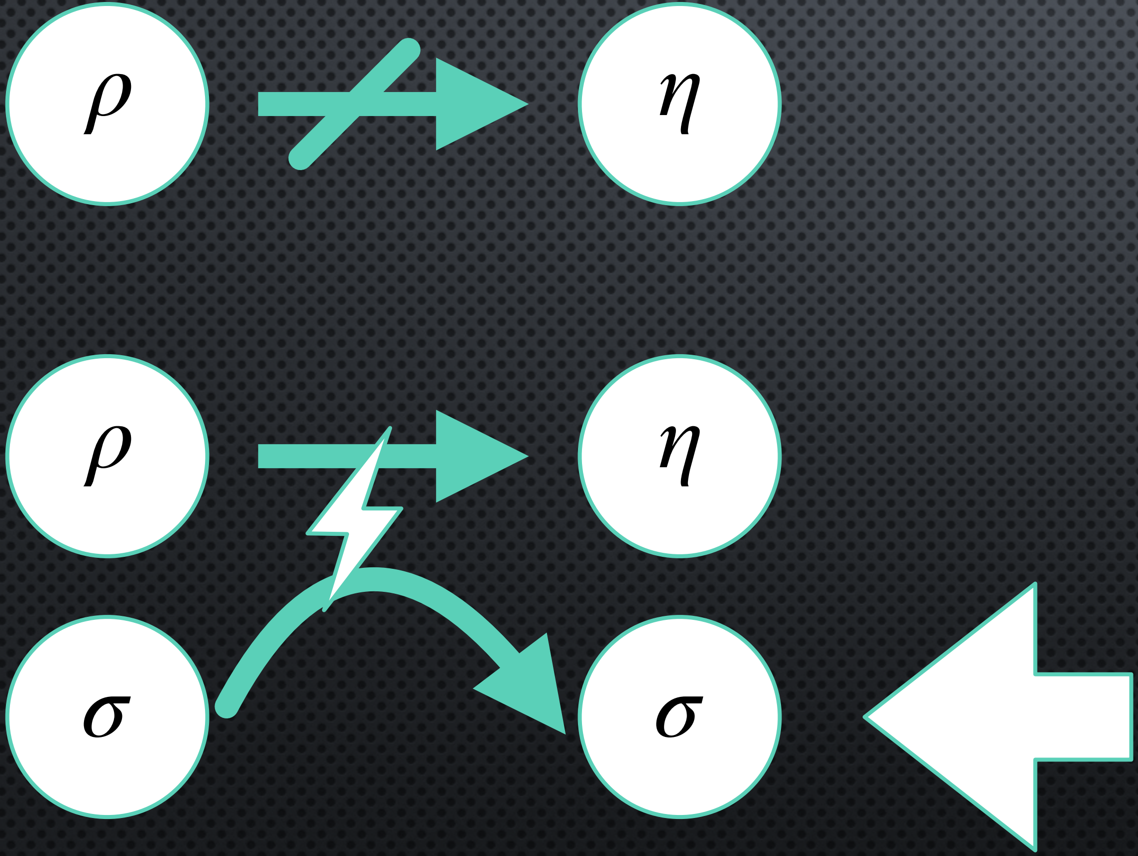
NANYANG QUANTUM HUB

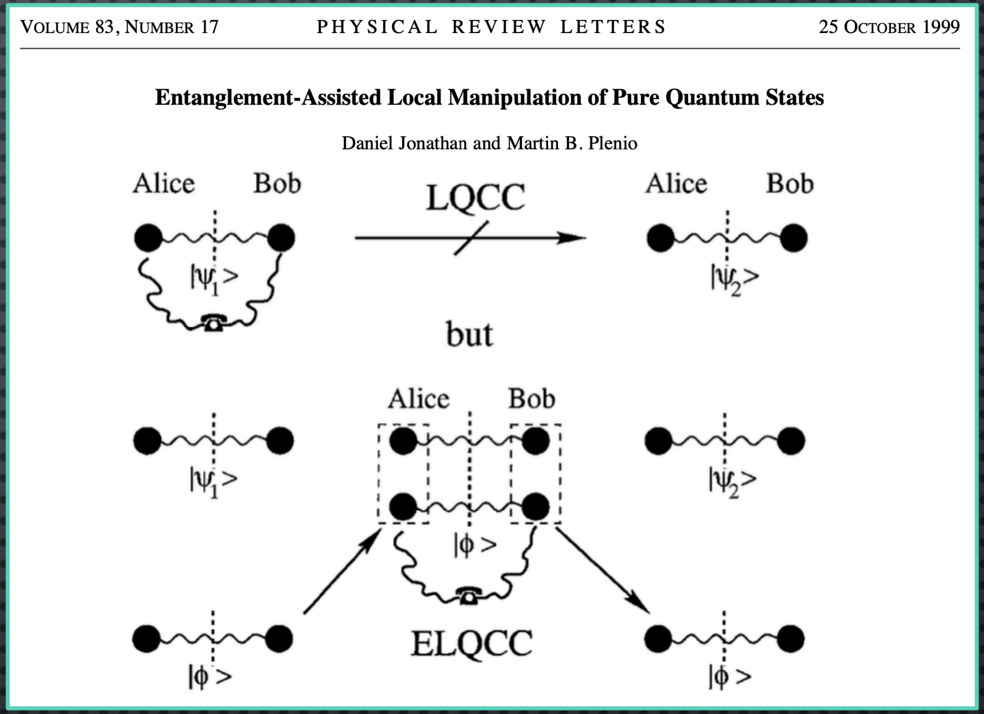
2 MOTIVATION



reusable!

3 MOTIVATION




VOLUME 83, NUMBER 17 PHYSICAL REVIEW LETTERS 25 OCTOBER 1999

Entanglement-Assisted Local Manipulation of Pure Quantum States

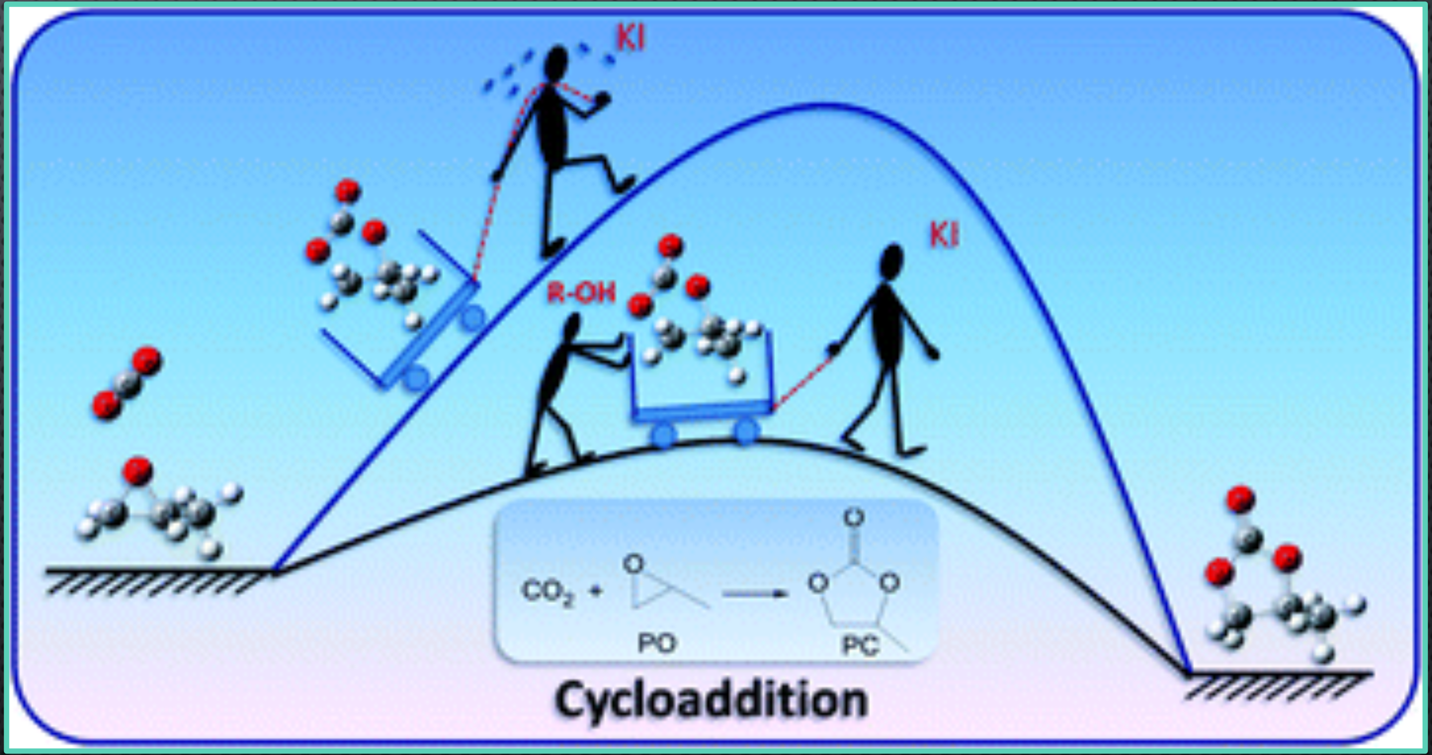
Daniel Jonathan and Martin B. Plenio

reusable!

MOTIVATION

Why does a catalyst help?

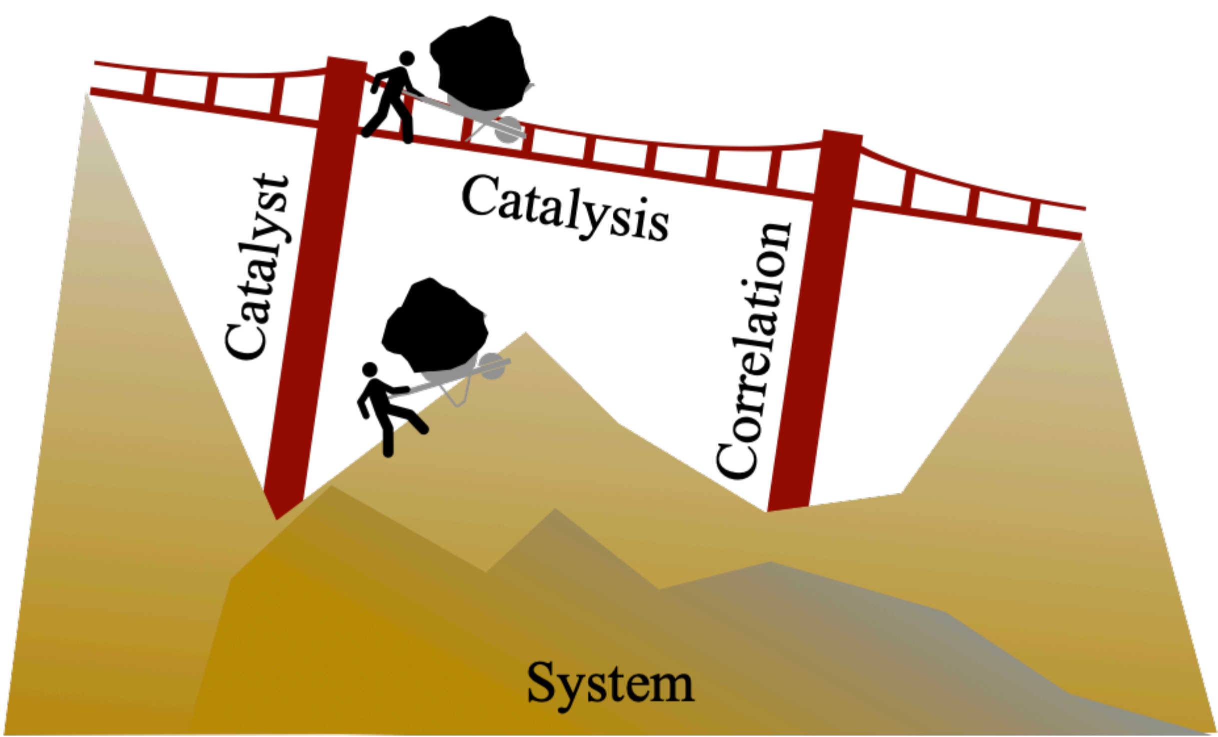
chemical catalysis

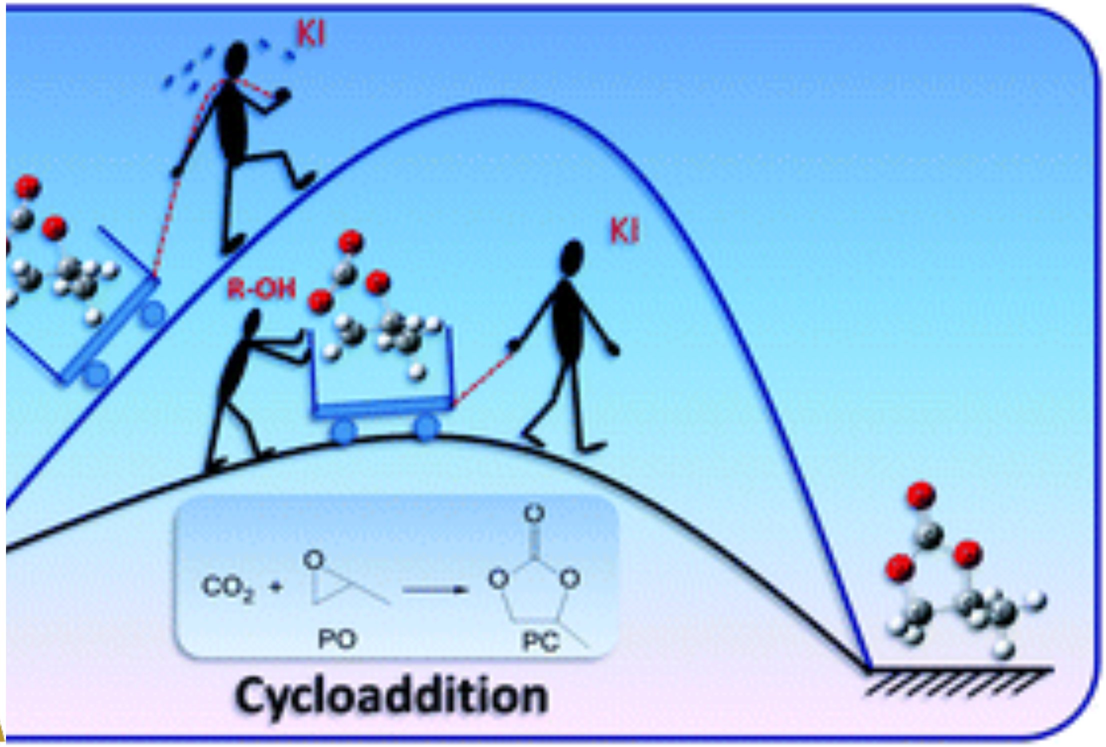


Green Chem., 2012, **14**, 2410

MOTIVATION

Why does a catalyst help?





Green Chem., 2012, **14**, 2410

CATALYSIS IN ACTION

VIA ELEMENTARY THERMAL OPERATIONS

MOTIVATION

CATALYSIS IN ACTION

1) understand what happens during catalysis

VIA ELEMENTARY THERMAL OPERATIONS

Lostaglio et al., Quantum **2**, 52 (2018)

2) develop more realistic catalytic processes

MOTIVATION: THERMAL OPERATIONS

$$\rho_S \xrightarrow{\text{TO}} \eta_S \Leftrightarrow \eta_S = \mathrm{Tr}_B \left[U_{SB} \left(\rho_S \otimes \tau_B^\beta \right) U_{SB}^\dagger \right]$$

where

- β: fixed ambient temperature
- H_S, H_B are fixed once chosen
- $\tau_B^\beta = e^{-\beta H_B}/Z$: Gibbs state w.r.t. β, H_B
- U_{SB}: global unitary s.t. $[U_{SB}, H_S + H_B] = 0$

MOTIVATION: THERMAL OPERATIONS

$$\eta_S = \mathrm{Tr}_B \left[U_{SB} \left(\rho_S \otimes \tau_B^{\beta} \right) U_{SB}^{\dagger} \right]$$

Needs complicated bath and unitary

→ hard to implement

Horodecki and Oppenheim, Nat. Comm. **4**, 2059 (2013)

Focus only on initial/final states

→ elusive dynamics

ELEMENTARY THERMAL OPERATIONS (ETO)

better implementability

TO

- hard to implement
- elusive dynamics

Lostaglio et al., Quantum **2**, 52 (2018)

Elementary Thermal Operations (ETO)

better characterization

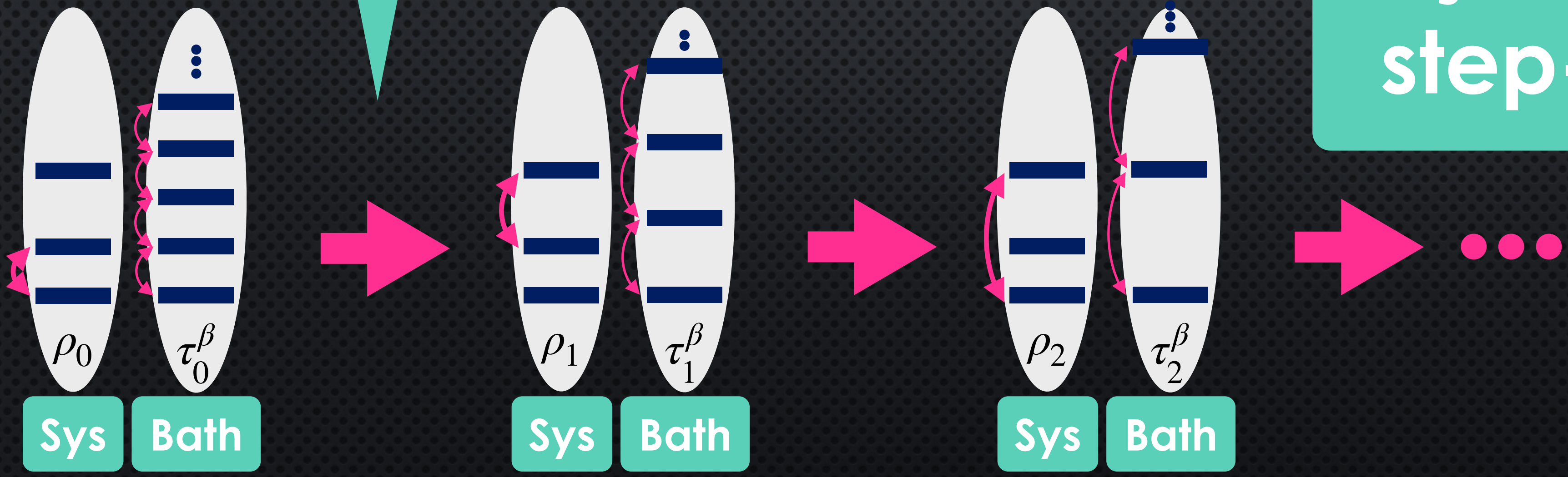
harmonic oscillator bath interacting w/ 2-lvls of system

ELEMENTARY THERMAL OPERATIONS (ETO)

intensity-dep. Jaynes-Cummings

$$H_{\text{int}} = g \sum_{n=1}^{\infty} \left(\sigma^{+} |n-1\rangle\langle n| + \sigma^{-} |n\rangle\langle n-1| \right)$$

Jaynes-Cummings type interaction



trajectory arises from step-wise structure

CATALYTIC ELEMENTARY THERMAL OPERATIONS (CETO)

Catalytic advantage w/

- straightforward recipe
- real-time trajectory

Catalytic ETO
(CETO)

CATALYST SETUP

- Small
  - qutrit sys $\otimes$ qubit cat = 6 dim. → manageable
- Exact
  - no correlation & error → most conservative

RESULTS

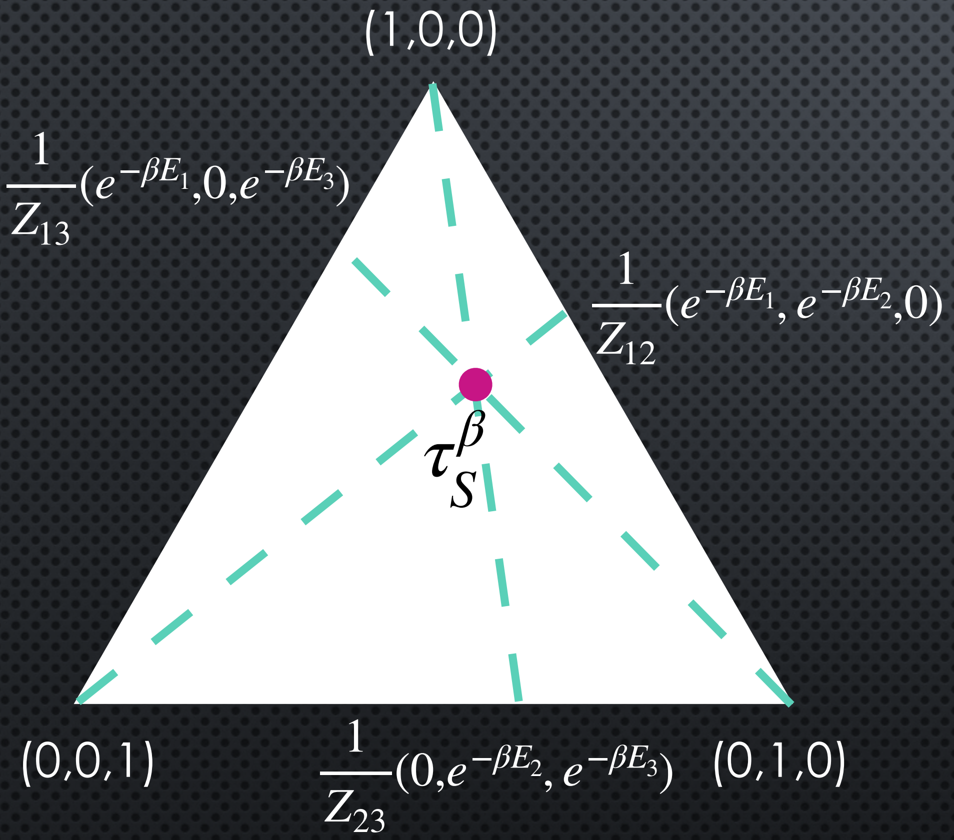
1. Useful qubit catalysis exists for ETO

2. Catalysts can be understood as temporary free energy storage

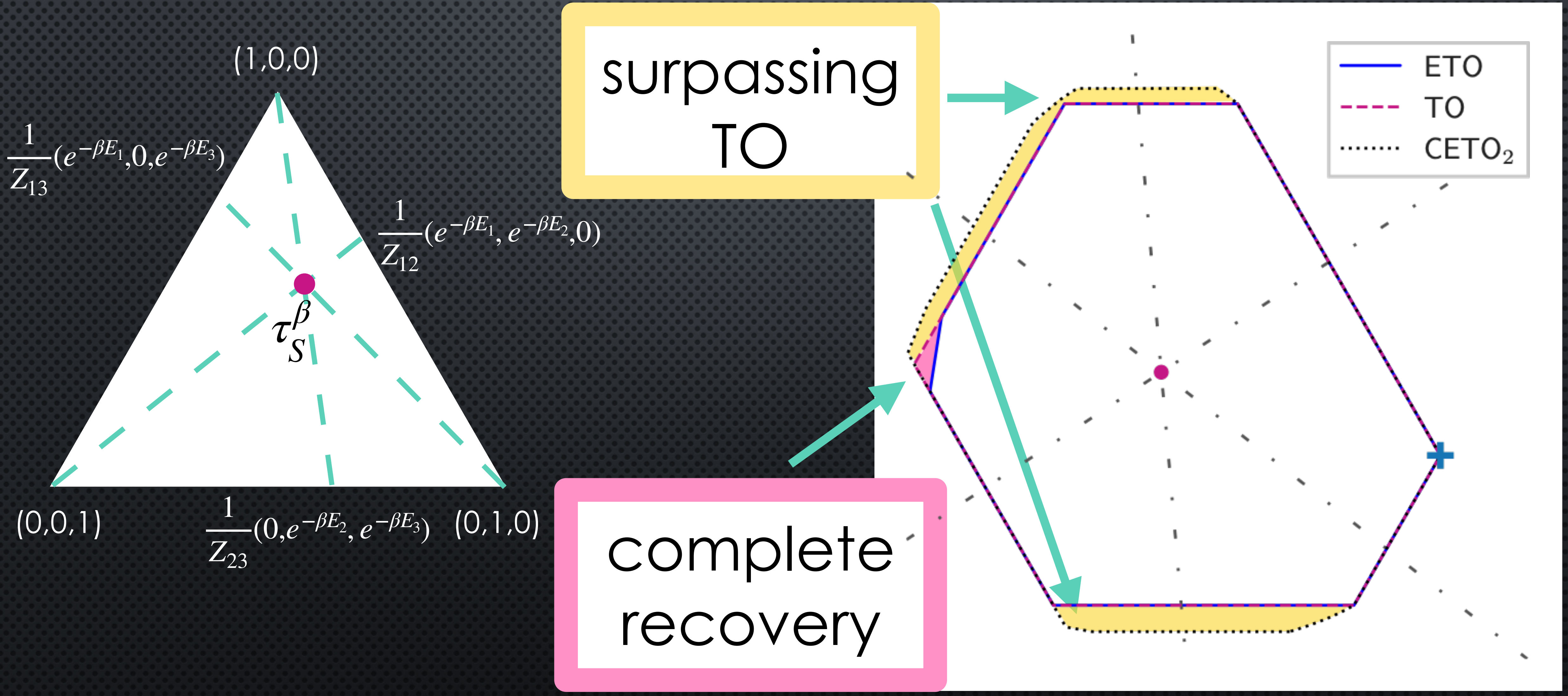
3. Existing computational cost for ETO is improved

* Consider only energy incoherent states

RESULT 1: USEFUL QUBIT CATALYSIS EXISTS



RESULT 1: USEFUL QUBIT CATALYSIS EXISTS



surpassing TO

complete recovery

RESULTS

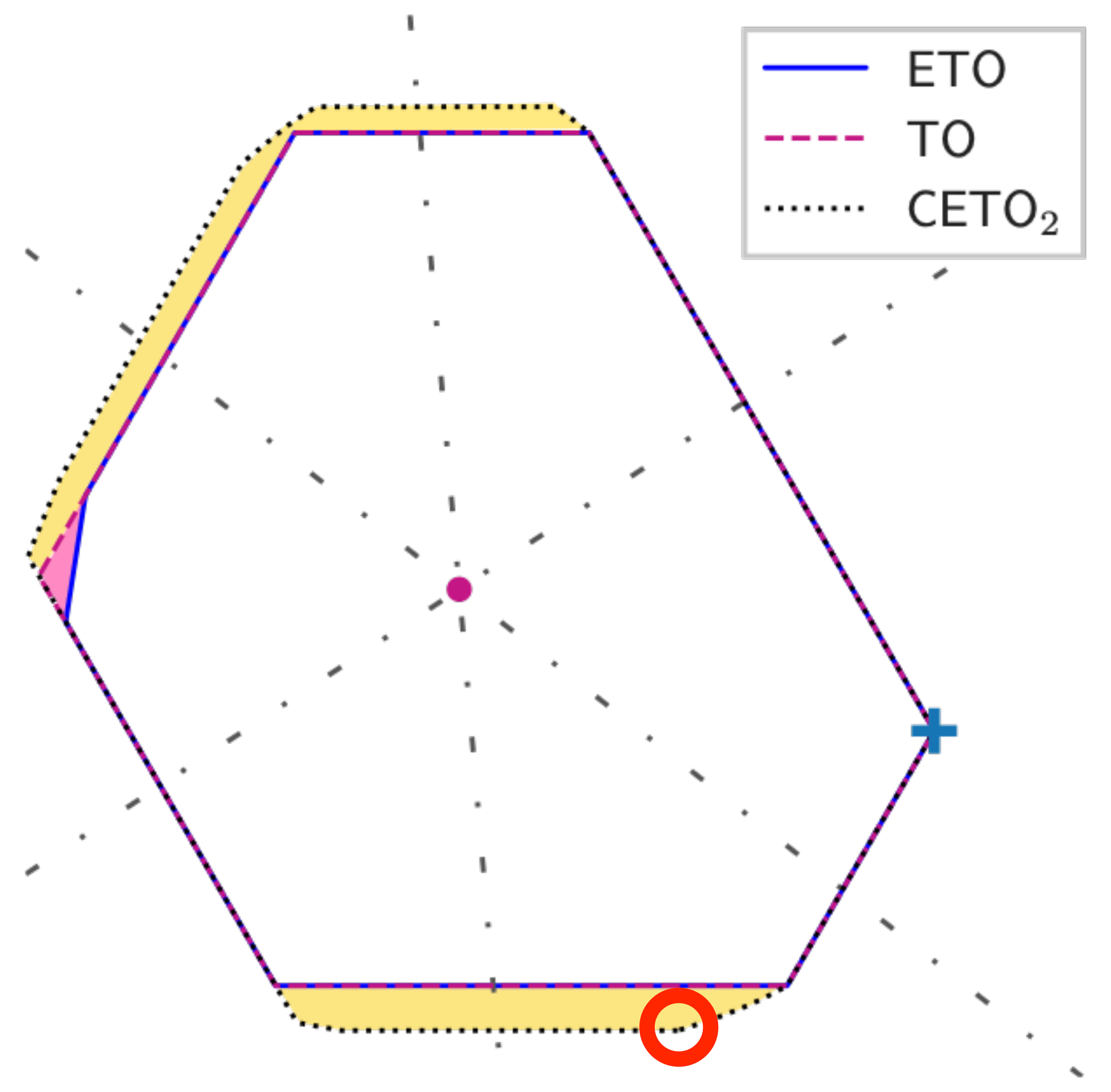
1. Useful qubit catalysis exists for ETO

2. **Catalysts can be understood as temporary free energy storage**

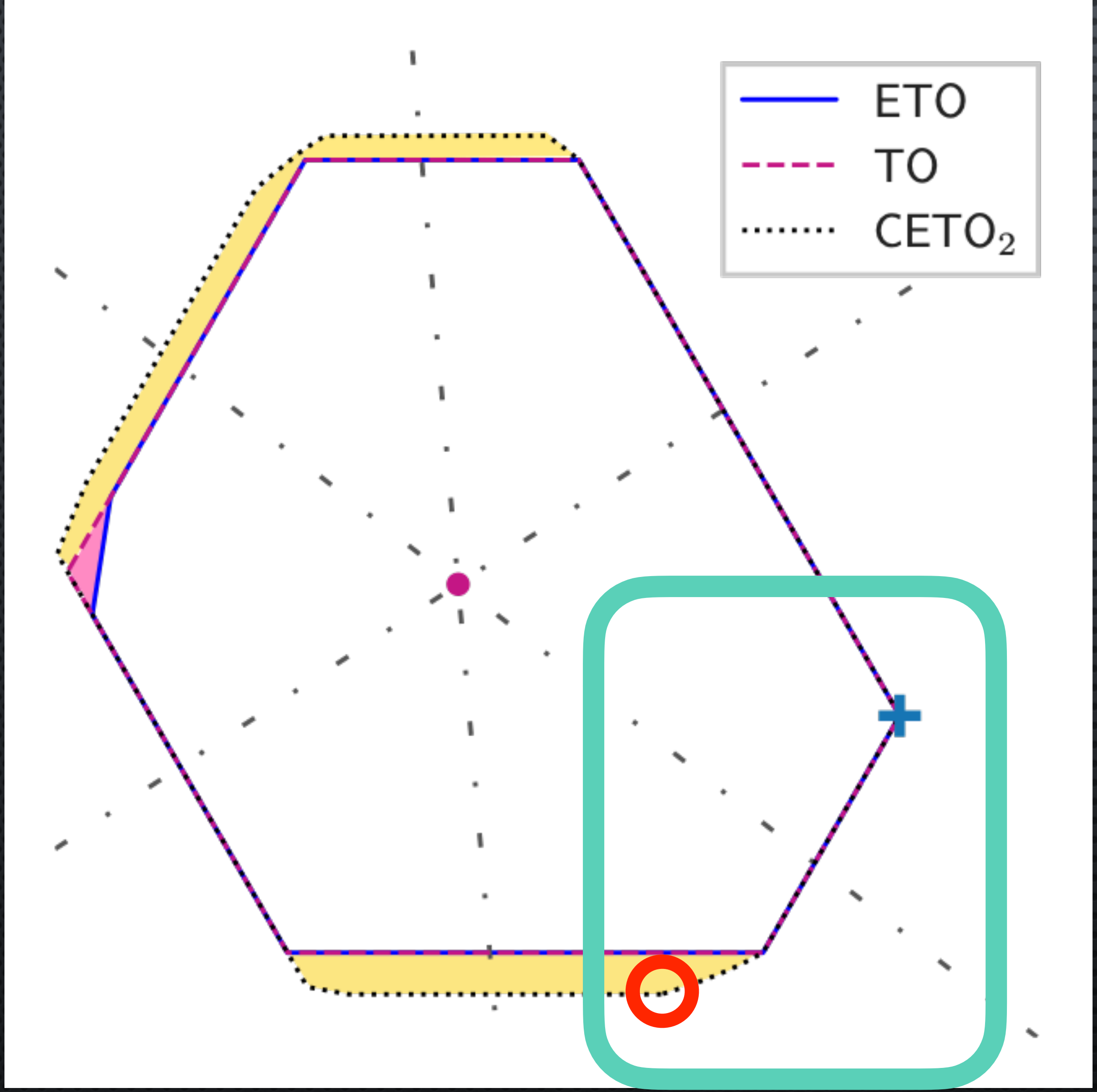
3. Existing computational cost for ETO is improved

*** Consider only energy incoherent states**

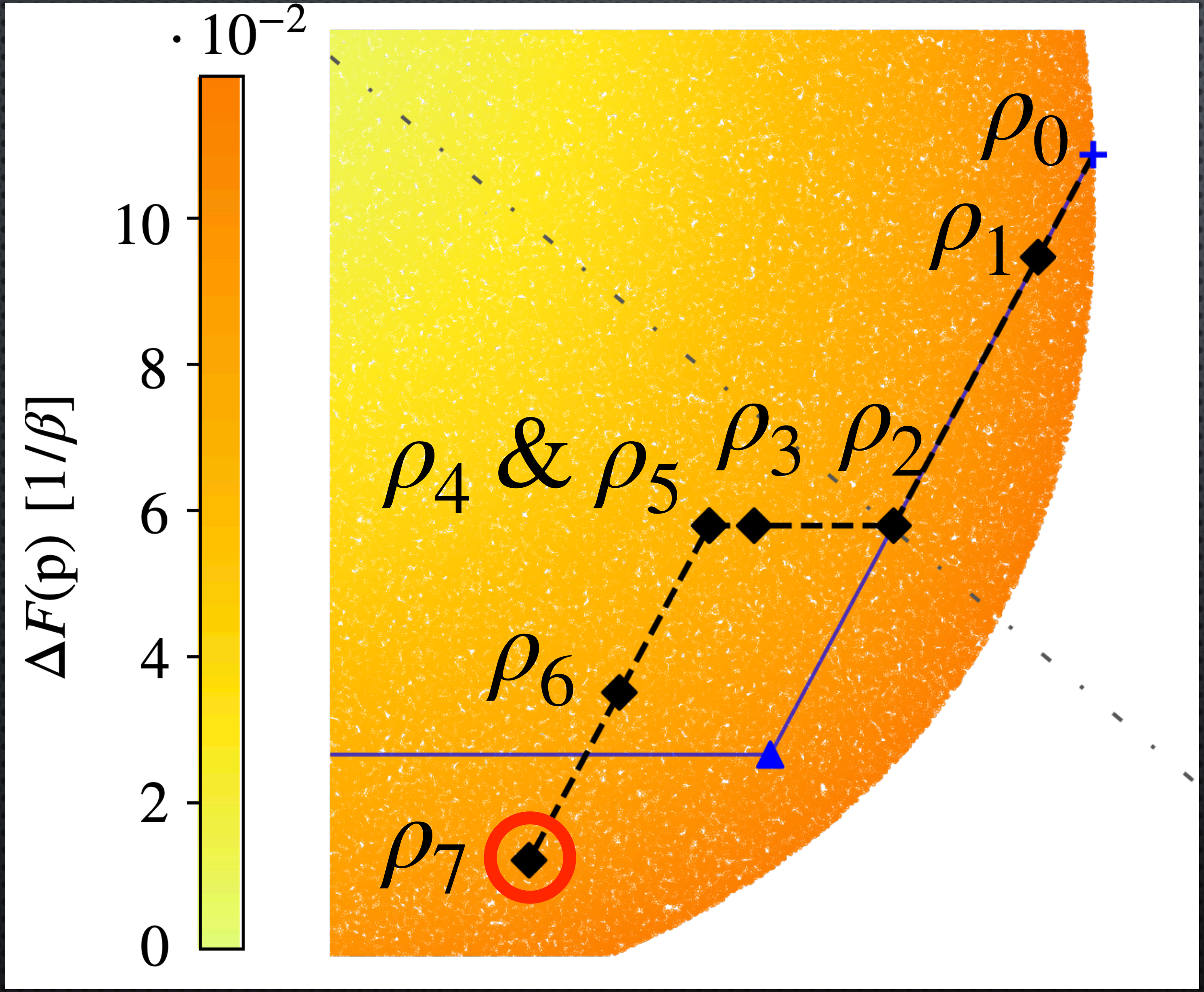
RESULT 2: TRACKING CATALYSIS — AN EXAMPLE



RESULT 2: TRACKING CATALYSIS — AN EXAMPLE

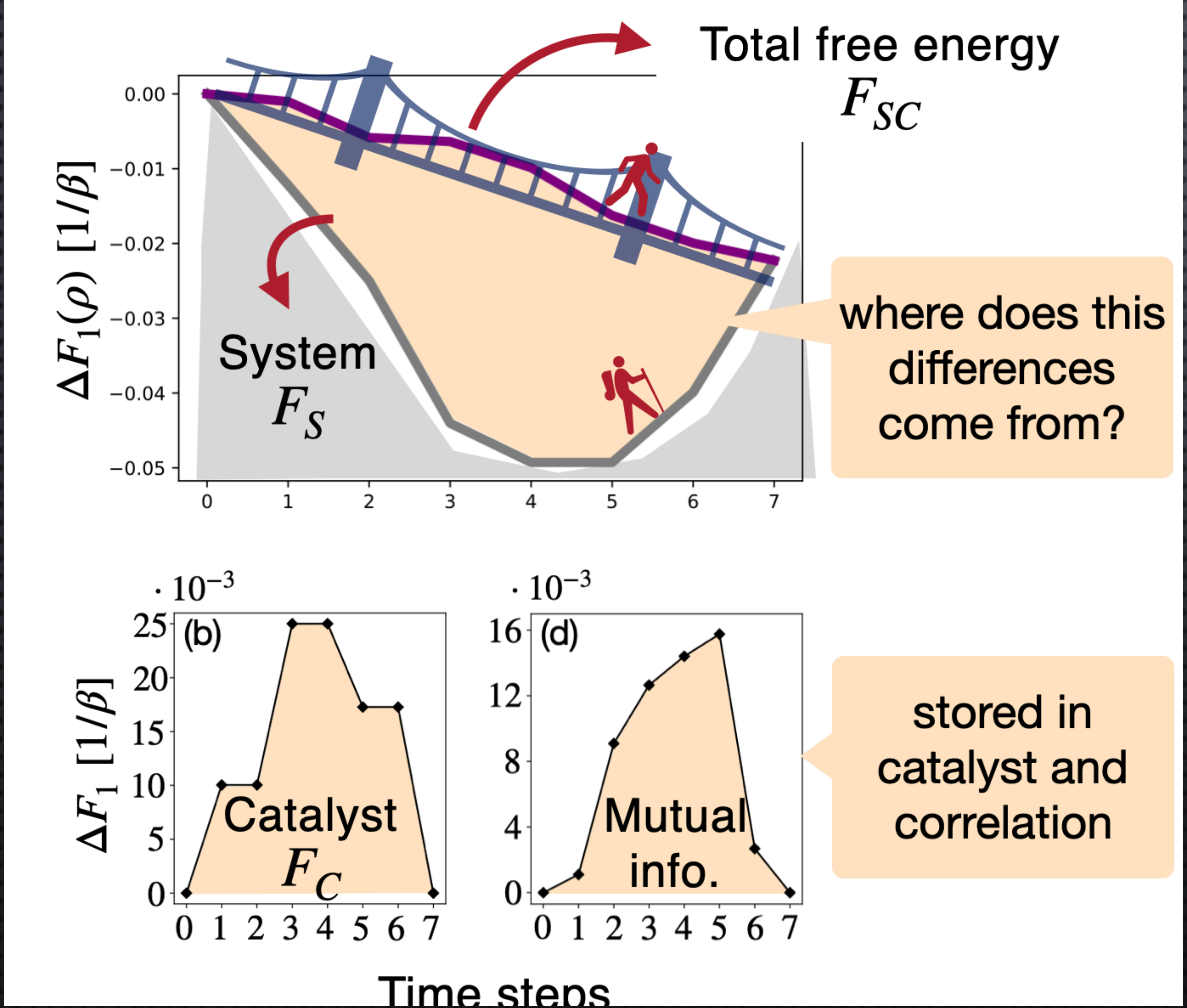


RESULT 2: TRACKING CATALYSIS — AN EXAMPLE



snapshots of system reduced states

RESULT 2: TRACKING CATALYSIS — AN EXAMPLE

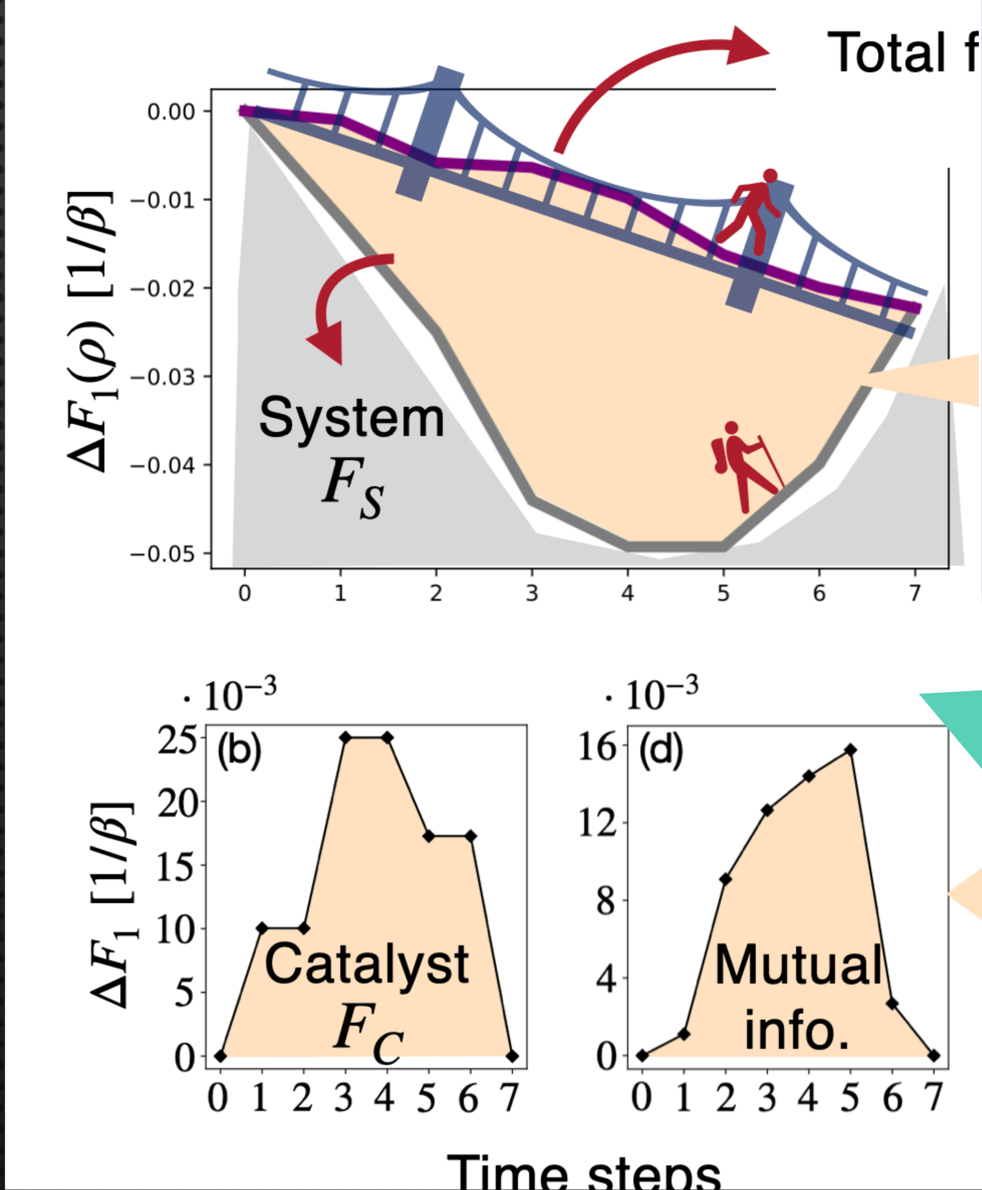


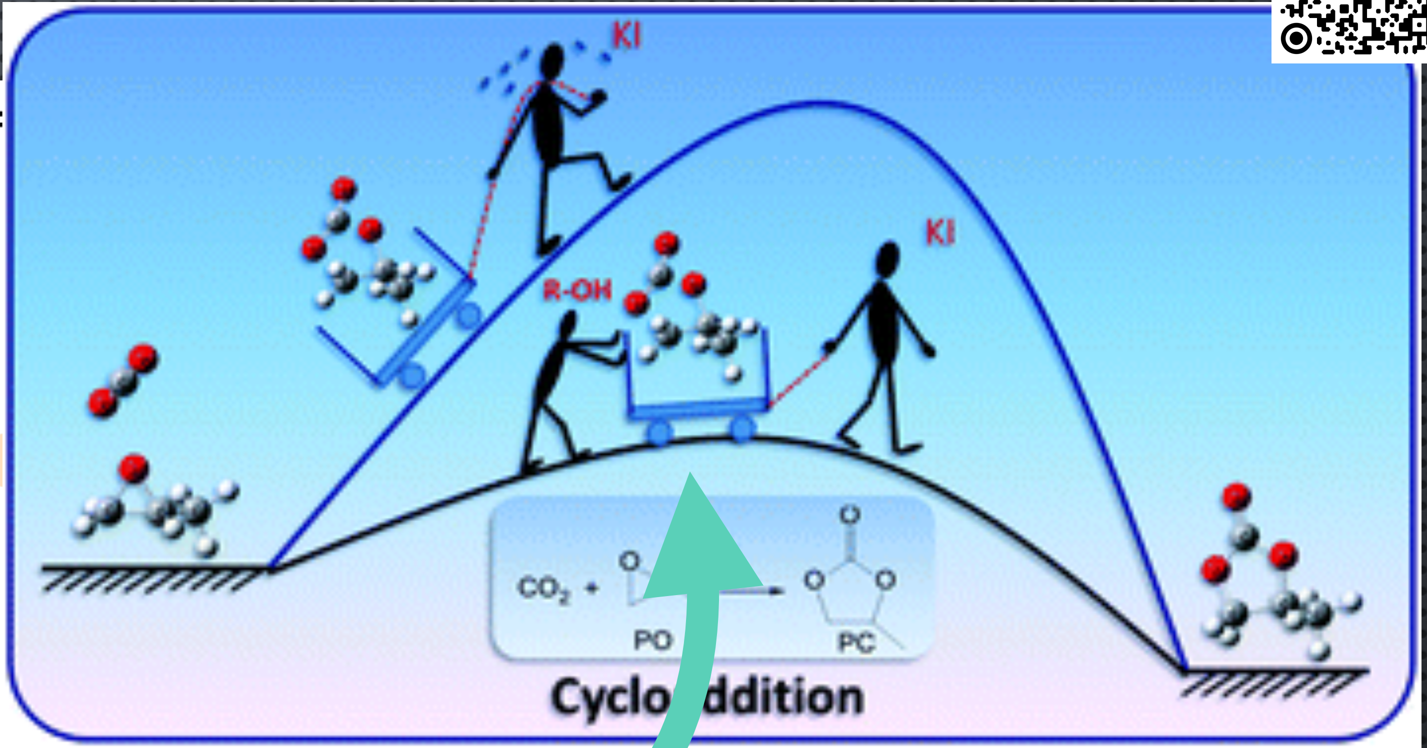
where does this differences come from?

stored in catalyst and correlation

catalysts as
“free energy storage”

RESULT 2: TRACKING CATALYSIS — AN EXAMPLE





Green Chem., 2012, **14**, 2410

stored in catalyst and correlation

Similar? 😉

RESULTS

1. Useful qubit catalysis exists for ETO
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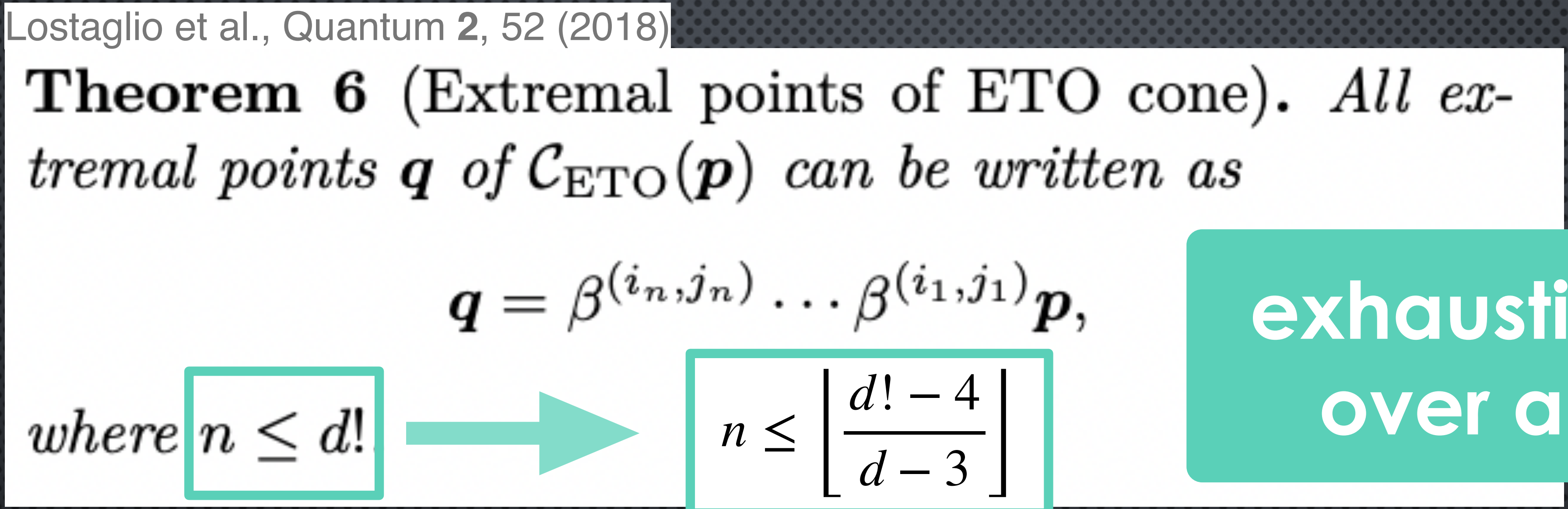
3. **Existing computational cost for ETO is improved**

*** Consider only energy incoherent states**

RESULT 3: TIGHTENING THE BOUND



Lostaglio et al., Quantum **2**, 52 (2018)

Theorem 6 (Extremal points of ETO cone). *All extremal points* $\boldsymbol{q}$ *of* $\mathcal{C}_{\mathrm{ETO}}(\boldsymbol{p})$ *can be written as*

$$\boldsymbol{q} = \beta^{(i_n,j_n)} \dots \beta^{(i_1,j_1)} \boldsymbol{p},$$

where $\boxed{n \leq d!} \rightarrow \boxed{n \leq \left\lfloor \frac{d!-4}{d-3} \right\rfloor}$

previous result

our result

exhaustive search over all such q

$\beta^{(i,j)}$: maximal swap between levels i and j

RESULT 3: TIGHTENING THE BOUND

polynomial scaling of $\ell_{\max}$ for special classes of initial states

$$n \leq \left\lfloor \frac{d! - 4}{d - 3} \right\rfloor$$

max. len.	Theory	Numerics
d=3	3 (proved separately)	3
d=4	20	8
d=5	58	16
d=6	238	23
d=7	1259	38

N: #ext. points

$N \sim 50$

$N \sim 700$

$N \sim 7 \times 10^3$

$N \sim 1.6 \times 10^6$

TAKE-HOME MESSAGES

- Simple catalytic process exists
- ETO is a nice playground to study catalysis
- Free energy evolution provides insights into the origin of catalytic power

And many more in our recent preprint!

arXiv:2209.15213