Testing quantumness without entanglement

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Outline

1. Motivation: testing quantumness of gravity
2. Mathematical formulation
3. A general bound
4. Application to systems of oscillators & main result

Motivation

What happens to the gravitational field of a mass placed in a spatial superposition?[1]

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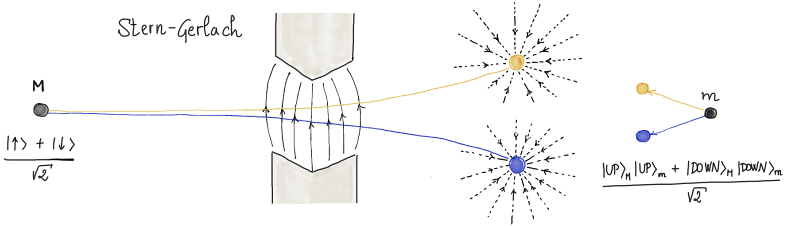

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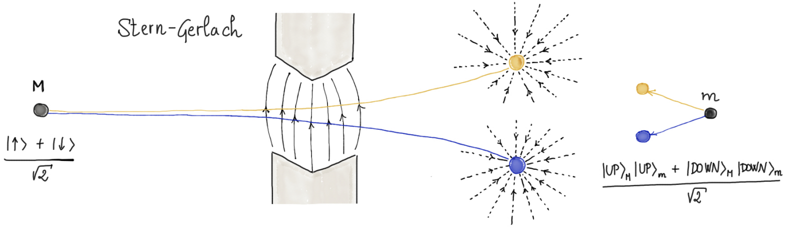

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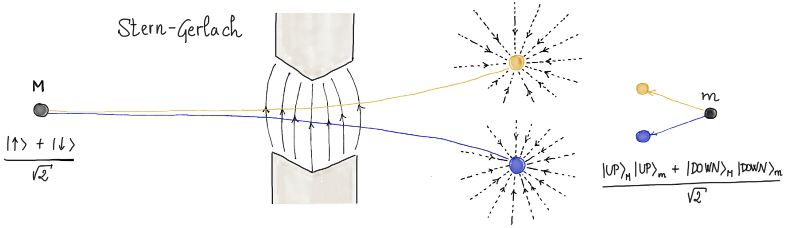

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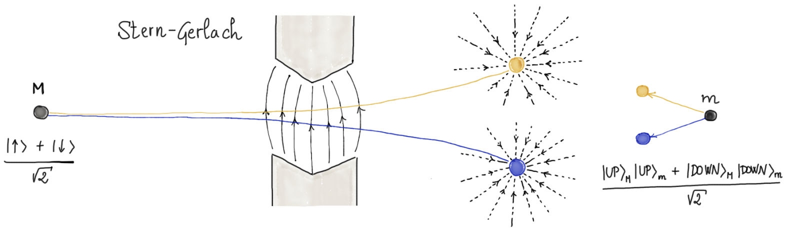

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Can we discriminate between these two options, experimentally?

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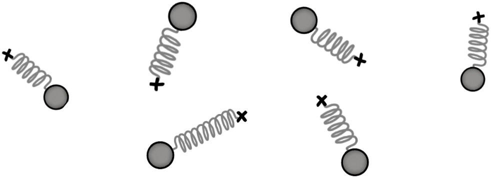

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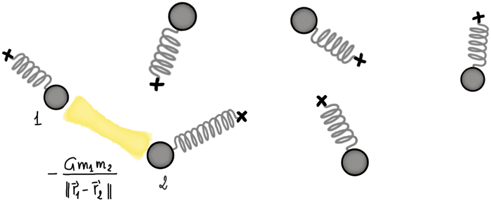

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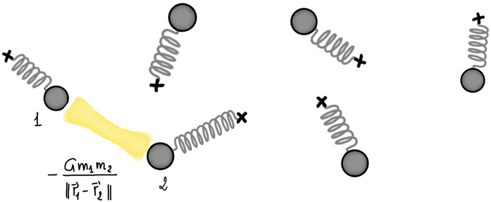

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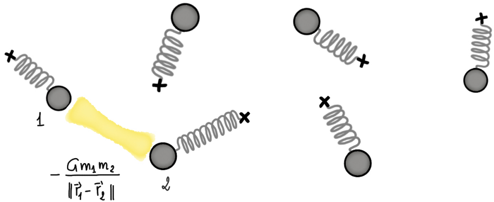

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1. Gravity acts as the unitary $U_G = e^{-iH_G t/\hbar}$, where[2]

$$H_G = \text{Newtonian Hamiltonian} = -\sum_{i<j} \frac{Gm_i m_j}{\|\vec{r}_i - \vec{r}_j\|}$$

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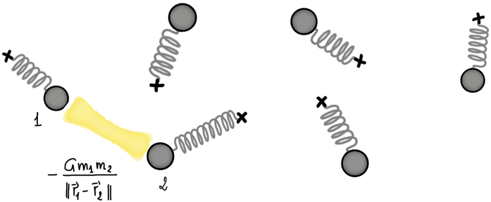

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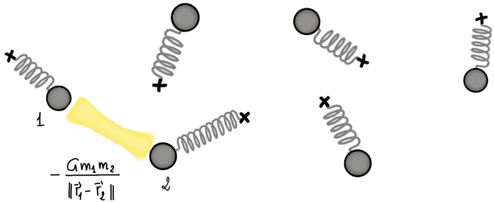

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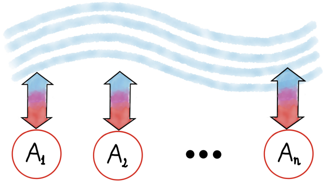

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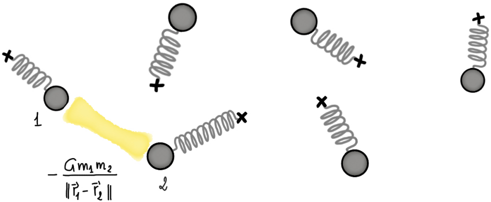

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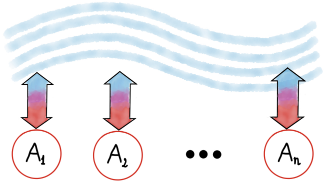

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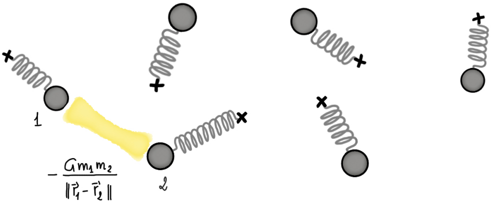

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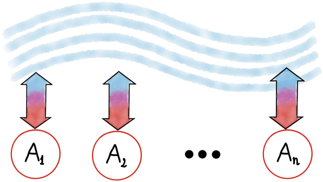

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(= local operations and classical communication)

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The problem

Main question

Given an isometry $U : A_1 \ldots A_n \to A'_1 \ldots A'_n$ on a multi-partite quantum system $1 : 2 : \ldots : n$, how well can it be simulated by means of LOCC?

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We can only prepare states from the ensemble $\mathscr{E} = \{p_\alpha, |\psi_\alpha\rangle\}_\alpha$
$\longrightarrow$ a good figure of merit is

$$F_{c\ell}(\mathscr{E}, U) := \sup_{\Lambda \in \mathrm{LOCC}(A \to A')} \sum_\alpha p_\alpha \operatorname{Tr}[\Lambda(\psi_\alpha)\psi_\alpha'],$$

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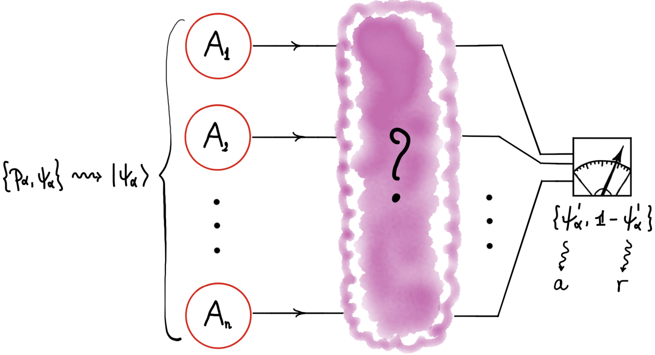
$\operatorname{Tr}[\Lambda(\psi_\alpha)\psi'_\alpha] =$ fidelity between simulated state $\Lambda(\psi_\alpha)$ and target ψ'_α.

Operational interpretation



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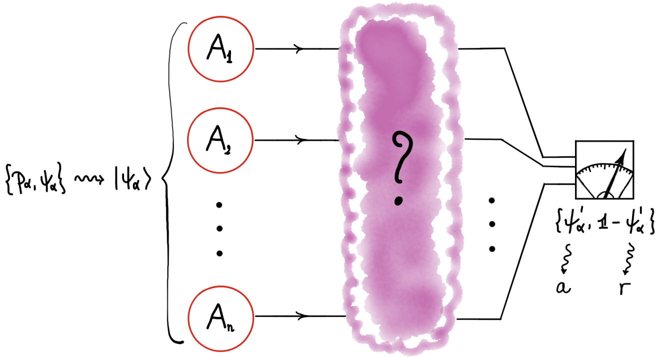

$$P(a|U) = \sum_{\alpha} p_\alpha \operatorname{Tr} \psi'_\alpha U \psi_\alpha U^\dagger = 1\,,$$

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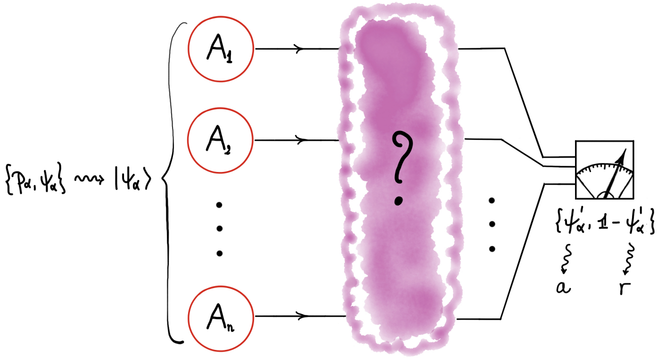

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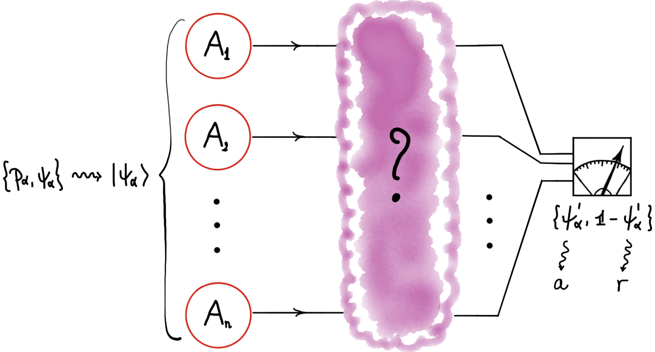

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If outcome a is recorded with frequency $> F_{c\ell}(\mathscr{E}, U)$, then  $\neq \mathrm{LOCC}$

A key tool

Theorem 1

For all ensembles $\mathscr{E} = \{p_\alpha, \psi_\alpha\}_\alpha$ and isometries $U : A \to A'$, it holds that

$$F_{c\ell}(\mathscr{E}, U) \leq \min_{J \subseteq [n]} \inf \left\{ \kappa > 0 : \; R_{AA'}^{\Gamma_J} \leq \kappa \, \omega_A \otimes \mathbb{1}_{A'}, \; \omega_A \in \mathcal{D}(\mathcal{H}_A) \right\},$$

$$R_{AA'} := \sum_\alpha p_\alpha \, (\psi_\alpha^*)_A \otimes (\psi'_\alpha)_{A'} \,,$$

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- Example: $n = 2,\ J = \{2\}$; then

$$\left(X_{A_1} \otimes Y_{A_2} \otimes W_{A'_1} \otimes Z_{A'_2} \right)^{\Gamma_J} = X_{A_1} \otimes Y_{A_2}^{\intercal} \otimes W_{A'_1} \otimes Z_{A'_2}^{\intercal} .$$

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- Alternative re-writing with conditional min-entropy:[3]

$$F_{c\ell}(\mathscr{E}, U) \leq \min_{J\subseteq[n]} \exp\left[-H_{\min}(A'|A)_{R^{\Gamma_J}}\right]$$

[3] Renner, PhD thesis (ETH Zürich, 2005), arXiv:quant-ph/0512258.

Proof.

$$F_{c\ell}(\mathscr{E}, U) = \sup_{\Lambda \in \mathrm{LOCC}} \sum_{\alpha} p_{\alpha} \langle \psi'_{\alpha} | \Lambda(\psi_{\alpha}) | \psi'_{\alpha} \rangle$$

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Application to a specific system

System of interest:
mechanical oscillators.

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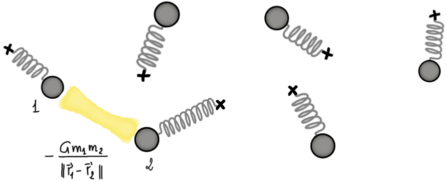

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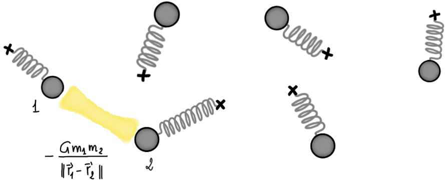

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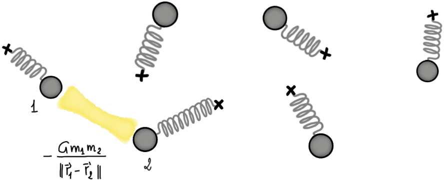

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- Canonical operators $r := (x_1, p_1, \ldots, x_n, p_n)^\intercal$. Commutation relations

$$[r, r^\intercal] = i\Omega\,, \qquad \Omega := \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^{\oplus n}.$$

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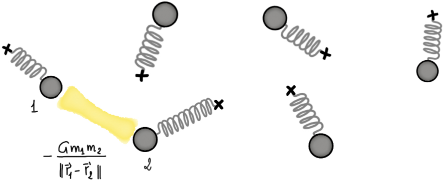

Each oscillator is 1-dim. Hilbert space $L^2(\mathbb{R})^{\otimes n} \simeq L^2(\mathbb{R}^n)$.

- Canonical operators $r := (x_1, p_1, \ldots, x_n, p_n)^\intercal$. Commutation relations

$$[r, r^\intercal] = i\Omega , \qquad \Omega := \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^{\oplus n} .$$

- **Coherent states** are 'easy' to prepare. Single mode:

$$\mathbb{C} \ni \alpha = \alpha_R + i\alpha_I \longrightarrow |\alpha\rangle := \exp\left[i\sqrt{2}\,(\alpha_I x - \alpha_R p)\right] |0\rangle \ .$$

Gaussian coherent state ensemble. $\lambda > 0$, fixed n: i.i.d. ensemble

$$\mathscr{E}_\lambda := \left\{ p_\lambda(\alpha)\, d^2\alpha,\ |\alpha\rangle\!\langle\alpha| \right\}^{\otimes n}_{\alpha\in\mathbb{C}}\,, \quad p_\lambda(\alpha) := \frac{\lambda}{\pi} e^{-\lambda|\alpha|^2}.$$

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- Definition #1:

$$U_{\mathrm{G}}^\dagger\, r\, U_{\mathrm{G}} = Sr + \delta\,;$$

S: $2n \times 2n$ real 'symplectic' matrix; $\delta \in \mathbb{R}^{2n}$.

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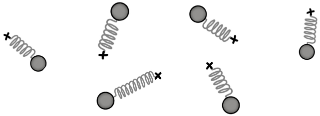
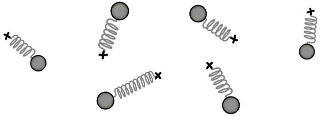
- Fact: *these two definitions are equivalent.*



Distance between oscillators $\gg$ oscillation amplitude



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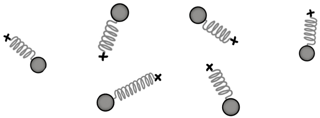

$\Longrightarrow$ Taylor expand $H_G = -\sum_{i<j} \frac{Gm_i m_j}{\|\vec{r}_i - \vec{r}_j\|}$ up to 2[th] order w.r.t. displacement of masses from equilibrium position.

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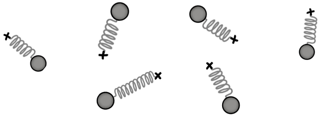

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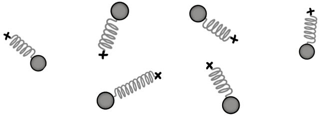

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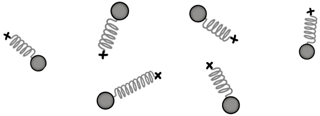

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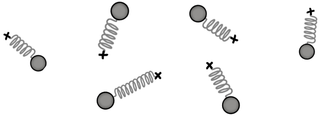

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Main result

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Gaussian i.i.d. ensemble $\mathscr{E}_\lambda$, $\lambda > 0$. Gaussian unitary U_{G} s.t. $U_{\mathrm{G}}^\dagger\, r\, U_{\mathrm{G}} = Sr + \delta$. Then

$$F_{c\ell}\left(\mathscr{E}_\lambda,\, U_{\mathrm{G}}\right) \leq f(\lambda, S) \coloneqq \min_{J\subseteq[n]} \frac{2^n (1+\lambda)^n}{\prod_{\ell=1}^{2n} \sqrt{2+\lambda+|z_\ell(\lambda,S,J)|}},$$

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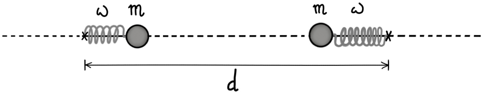
- Nevertheless, $F_{\mathcal{C}\ell}(\mathscr{E}_\lambda, U_{\mathrm{G}}) < 1$! Processes mapping product states to product states can be very far from LOCC (e.g. swap).

Recap & example

Simplest example:
two oscillators on a line.



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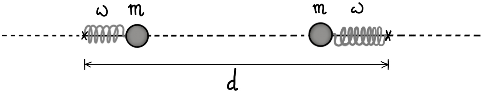

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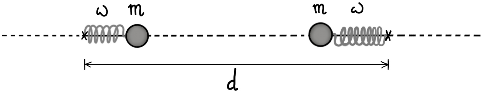

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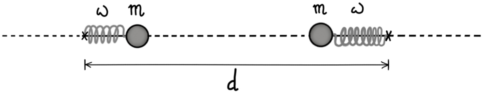

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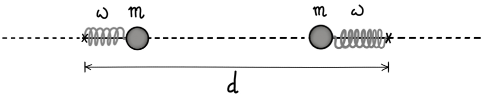

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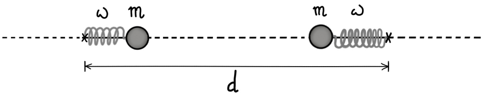

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⚠ Several assumptions & approximations.

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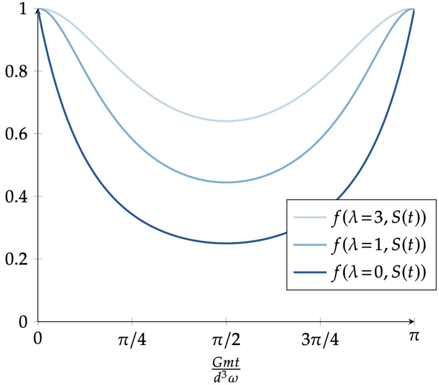

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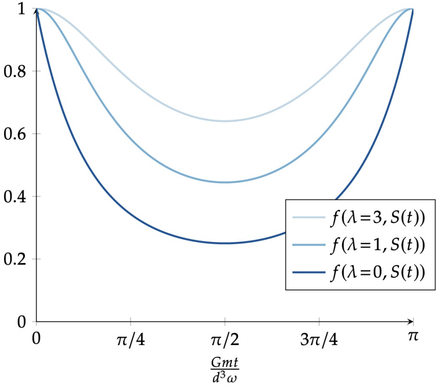

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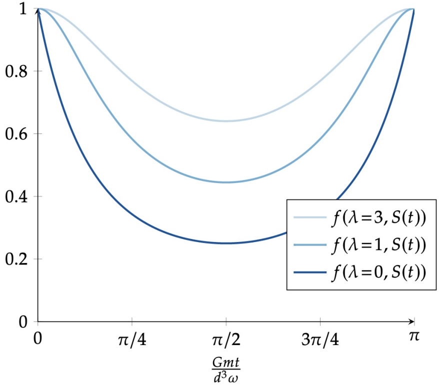

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$$f(0, S(t)) = \frac{1}{1 + \left|\sin\left(\frac{Gmt}{d^3\omega}\right)\right|} .$$

- Ideally, wait t_0 s.t. $\frac{Gmt_0}{d^3\omega} = \frac{\pi}{2} \implies f$ is at a minimum. In practice, $t_0 \sim 3\,\mathrm{d}$...

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[4] Fein et al., *Nat. Phys.* **15**:1242, 2019.

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LIGO's suspended mirror, $m \sim 10\,\text{kg}$.

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Thank you!