Resource theories of communication and coherent routing of information

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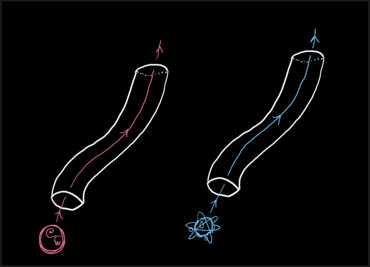
Resource theories of communication
New Journal of Physics 22, 073014 (2020) &
Quantum networks with coherent routing of information through multiple nodes
arXiv:2208.00480 (2022)

Outline

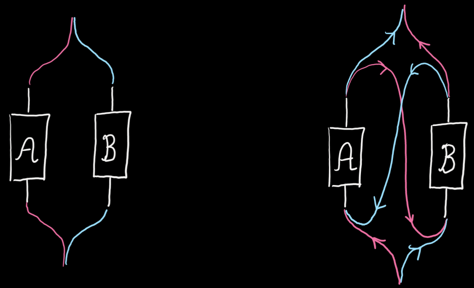
From classical to quantum Shannon theory

- Messages encoded in bits $\rightarrow$ Messages encoded in qubits
- Gives rise to many new possibilities, such as perfectly secure communication without pre-established keys.



A second-quantised Shannon theory

- Classical configuration of transmission lines → Quantum configuration of transmission lines
  - Coherent control over the paths of information carriers "coherent routing"
  - Coherent control over the order of communication channels "quantum switch"
- Both paradigms have been shown to enable enhancements in the classical and quantum capacity of noisy channels.



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S. Salek et al, arXiv: 1809.06655 (2018)
G. Chiribella et al, New J. Phys. (2021)
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Key questions

- How should these advantages be quantified and compared?
- What are the extra ingredients that give rise to these advantages?
- What are the criteria for a new model to be meaningful?
  - Can we also do coherent control over encoding operations?

- To answer this: we formulate a general framework for **resource theories of communication**.
  - Quantifies the differences between different forms of coherent control.
  - Provides a general framework to define new paradigms of communication, including a minimal condition that all communication models must satisfy.

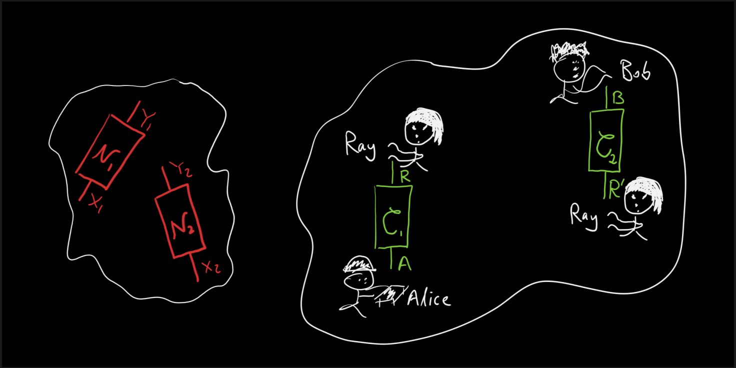
Resource theories of communication

- Resources: quantum channels
- Possible transformations: all valid supermaps $\mathbb{M}$
- Free transformations: a restricted choice of supermaps $\mathbb{M}_{\text{free}}$.

- Main question: what are the reasonable ways to combine quantum channels in a communication settings?
- Start with standard quantum Shannon theory: parallel and sequential use of channels between a sender and receiver, with local encoding/decoding operations.

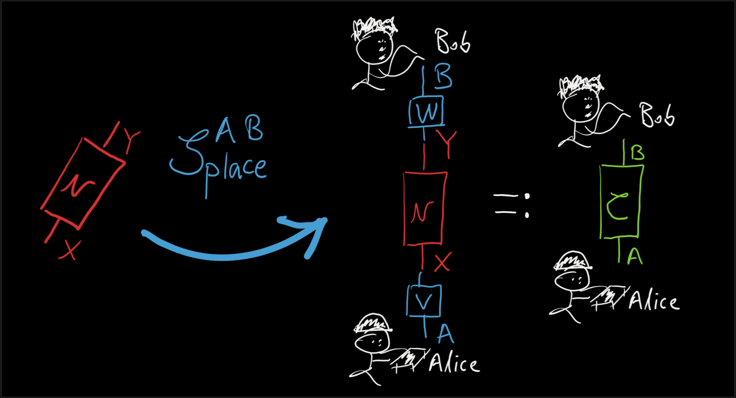
Placed and unplaced channels

- We consider two types of initial resources: placed and unplaced channels.
- **Unplaced channels** represent abstract communication devices, where the input and output systems correspond to abstract Hilbert spaces.
- **Placed channels** represent communication devices that have been placed between a sender and a receiver, where the input and output systems correspond to specific locations in spacetime.



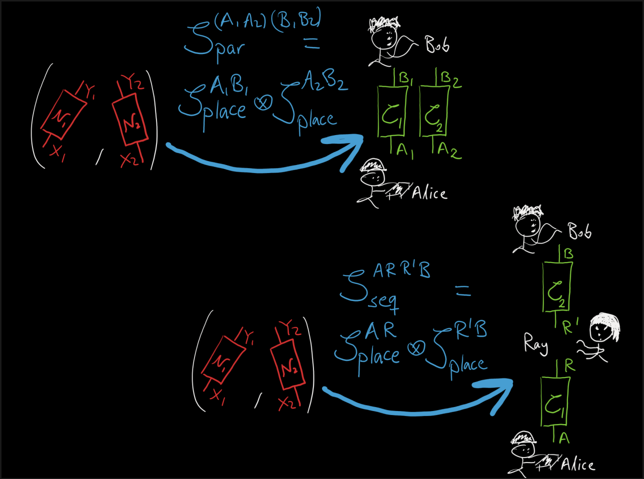
Basic placement supermap

- Our first free operation is the **basic placement supermap**, which maps an unplaced channels to a placed channel.
- Should be thought of as being performed by the communication provider.



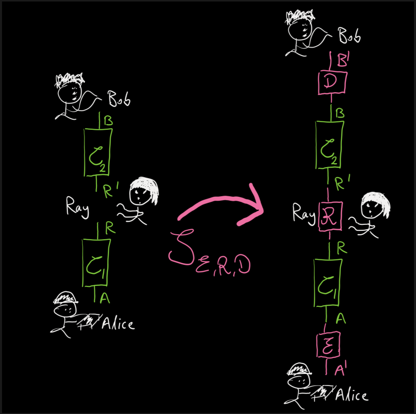
Parallel and sequential placement supermaps

- In general, **placement supermaps** are any supermaps from unplaced to placed channels.
- Parallel and sequential placements are constructed using a tensor product of multiple basic placement supermaps.



Party supermap

- Our second type of free operation is **party supermaps**, which map a set of placed channels between any number of parties to a single placed channel between a sender and receiver.
- Should be thought of as being performed by the communicating parties.
- For example, the encoding-repeater-decoding supermap:



General resource theories of communication

Definition

A **resource theory of communication** is specified by a set of free supermaps $\mathbb{M}_{\mathrm{free}}$ containing (a) placement supermaps and (b) party supermaps.

- Standard quantum Shannon theory is generated by the free supermaps:
  1. basic placement supermap
  2. encoding-repeater-decoding party supermap
- In principle, any choice of free supermaps defines a resource theory of communication, but it need not be a meaningful one.

Side-channel generating operations

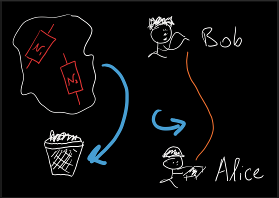
- A meaningful resource theory of communication should not allow the sender and receiver to communicate independently of the communication devices from which their communication protocol is built:

Definition

(Side-channel generating operations.) A supermap $\mathcal{S} \in \mathrm{M}$ *generates a classical (quantum) side-channel* if there exist two free supermaps $\mathcal{S}_1 \in \mathrm{M}_{\mathrm{free}}$ and $\mathcal{S}_2 \in \mathrm{M}_{\mathrm{free}}$ such that, for all choices of input channels $(\mathcal{N}_1, \ldots, \mathcal{N}_k)$ for supermap $\mathcal{S}_1$, one has

$$(\mathcal{S}_2 \circ \mathcal{S} \circ \mathcal{S}_1)(\mathcal{N}_1, \ldots, \mathcal{N}_k) = \mathcal{C}\,,$$

where $\mathcal{C}$ is a fixed placed quantum channel with non-zero classical (quantum) capacity.

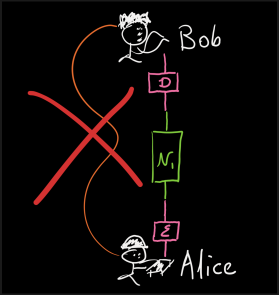


Minimal condition: no side-channel generation

Condition

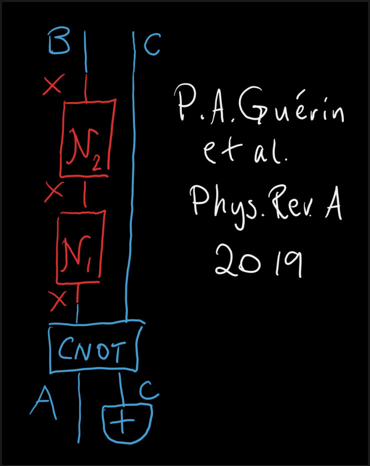
(No Side-Channel Generation.) In a resource theory of classical (quantum) communication, no free operation $\mathcal{S} \in \mathsf{M}_{\mathrm{free}}$ *should generate a classical (quantum) side-channel.*

- Just a minimal condition: not every resource theory satisfying the condition is necessarily a meaningful one.



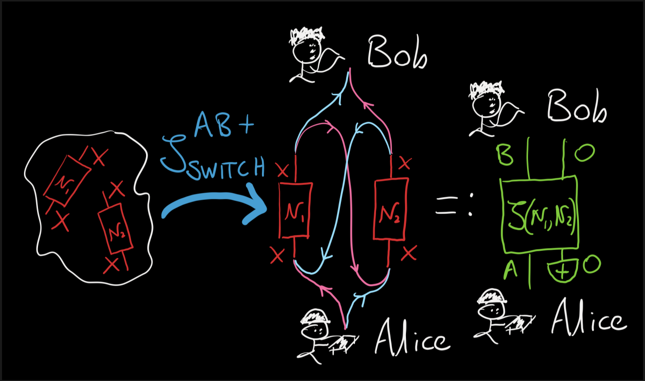
Not allowed: coherent control over encoding operations

- An example of a communication protocol that violates the minimal condition.



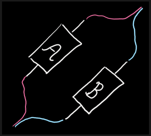
Quantum Shannon theory with coherent control over causal orders

- Quantum Shannon theory with coherent control over causal orders is generated by the free supermaps:
  1. basic placement supermap
  2. encoding-repeater-decoding party supermap
  3. quantum switch placement supermap
- Satisfies the minimal requirement.



Coherent routing of information through alternative paths

- Quantum physics enables coherent routing of information through a superposition of paths through two different channels.[1][2]
- However, this cannot be described as a supermap on the original channels alone.[3]



- Physically: superposition of sending a particle through one path with the vacuum in the other, and vice versa.[4][5]
  - When the particle is transmitted, the device will affect the particle's **internal degrees of freedom** (d.o.f.), e.g. photon polarisation (described by Hilbert space $\mathcal{H}$).
  - When the particle is not sent, the device will act on the (orthogonal) **vacuum state** $|\mathrm{vac}\rangle$ (which is the unique state in the vacuum Hilbert space $\mathcal{H}_{\mathrm{Vac}}$).

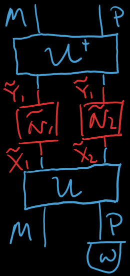
[1] D. K. L. Oi, *Physical Review Letters* **91**, 067902 (2003).
[2] N. Gisin *et al.*, *Physical Review A* **72**, 012338 (2005).
[3] A. Soeda, *Int. Conf. on Quant. Inf. and Tech. (talk)* (2013).
[4] X.-Q. Zhou *et al.*, *Nature Communications* **2**, 1–8 (2011).
[5] G. Chiribella, H K, *Proc. R. Soc. A* **475**, 20180903 (2019).

Vacuum extension of a channel

- The action of a communication channel is specified by a **vacuum extension** $\widetilde{\mathcal{N}} : \mathcal{L}(\mathcal{H}_X \oplus \mathcal{H}_{\text{Vac}}) \to \mathcal{L}(\mathcal{H}_Y \oplus \mathcal{H}_{\text{Vac}})$ of the original quantum channel[6,7] $\mathcal{N}$ satisfying

$$\begin{cases} \widetilde{\mathcal{N}}(\rho) = \mathcal{N}(\rho) & \forall \rho \in \mathcal{L}(\mathcal{H}_X) \\ \widetilde{\mathcal{N}}(|\text{vac}\rangle\langle\text{vac}|) = |\text{vac}\rangle\langle\text{vac}| \, . \end{cases}$$

- With two vacuum-extended channels $\mathcal{N}_1$ and $\mathcal{N}_2$, we can construct the coherent control over the two channels:



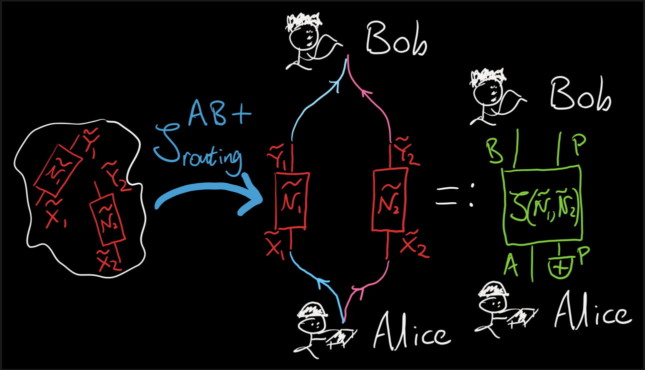
$$\mathcal{U} : \begin{cases} |\psi\rangle_M \otimes |0\rangle_P = |\psi\rangle_{\tilde{X}_1} \otimes |\text{vac}\rangle_{\tilde{X}_2} \\ |\psi\rangle_M \otimes |1\rangle_P = |\text{vac}\rangle_{\tilde{X}_1} \otimes |\psi\rangle_{\tilde{X}_2} \end{cases}$$

[G. Chiribella & H.K., Proc. R. Soc. A, 2019]

[6] G. Chiribella, H K, *Proc. R. Soc. A* **475**, 20180903 (2019).
[7] J. Åberg, *Annals of Physics* **313**, 326–367 (2004).

Quantum Shannon theory with coherent control over trajectories

- Quantum Shannon theory with coherent control over trajectories is generated by the free supermaps:
    1. basic placement supermap
    2. encoding-repeater-decoding party supermap
    3. coherent routing placement supermap
- Satisfies the minimal requirement.



Coherent routing over a network of noisy channels

- Transmission between two nodes is described by a binary asymmetric channel

$$\mathcal{E}(\rho) = p\,|0\rangle\langle 0| + (1-p)\,\rho_{\mathrm{diag}}\,. \tag{1}$$

- After n nodes, the probability of successful transmission exponentially decays:

$$\mathcal{E}^n(\rho) = [1-(1-p)^n]\,|0\rangle\langle 0| + (1-p)^n \rho_{\mathrm{diag}}\,. \tag{2}$$

- Coherent control over two sequences of n identical channels $\widetilde{\mathcal{E}}$:

$$\mathcal{S}^{|+\rangle\langle+|}_{\widetilde{\mathcal{E}}^n,\widetilde{\mathcal{E}}^n}(\rho) = \frac{\mathcal{E}^n(\rho) + F^n \rho F^{\dagger\,n}}{2} \otimes |+\rangle\langle+|_P + \frac{\mathcal{E}^n(\rho) - F^n \rho F^{\dagger\,n}}{2} \otimes |-\rangle\langle-|_P \tag{3}$$

  where the **vacuum interference operator** $F := \sum_i \bar{\alpha}_i\, E_i\,.$

- If F^n is non-zero, can transmit classical information even if $\mathcal{E}$ is completely noisy! For large n require $||F||_\infty = 1$.
- For a natural realisation of the BAC, we have $F = |0\rangle\langle 0|$:

$$\mathcal{S}^{|+\rangle\langle+|}_{\infty}(\rho) = |0\rangle\langle 0| \otimes (q_+\,|+\rangle\langle+| + q_-\,|-\rangle\langle-|)\,, \tag{4}$$

  with $q_\pm := (1 \pm \langle 0|\rho|0\rangle)/2$, which has classical capacity $\log_2(5/4) \approx 0.32$.

Summary and outlook

- Resource theory of communication
  1. placed/unplaced channels
  2. placement/party supermaps
- Minimal condition: no side-channel generation
  1. coherent control over causal orders ✓
  2. coherent control over trajectories ✓
  3. coherent control over encoding operations $\times$
- Quantifies the differences between new communication models and their potential advantages
- Novel type of advantage from quantum Shannon theory with coherent control over trajectories: can suppress the exponential decay of successful transmission through an asymptotically long sequence of channels.