QChannel RResource Marginal Problems

University of BRISTOL

Chung-Yun Hsieh

ICFO

Q Channel

R esource

Marginal Problems

C-Y Hsieh, M Lostaglio, A Acín, *Phys. Rev. Research* **4**, 013249 (2022)

Q Channel

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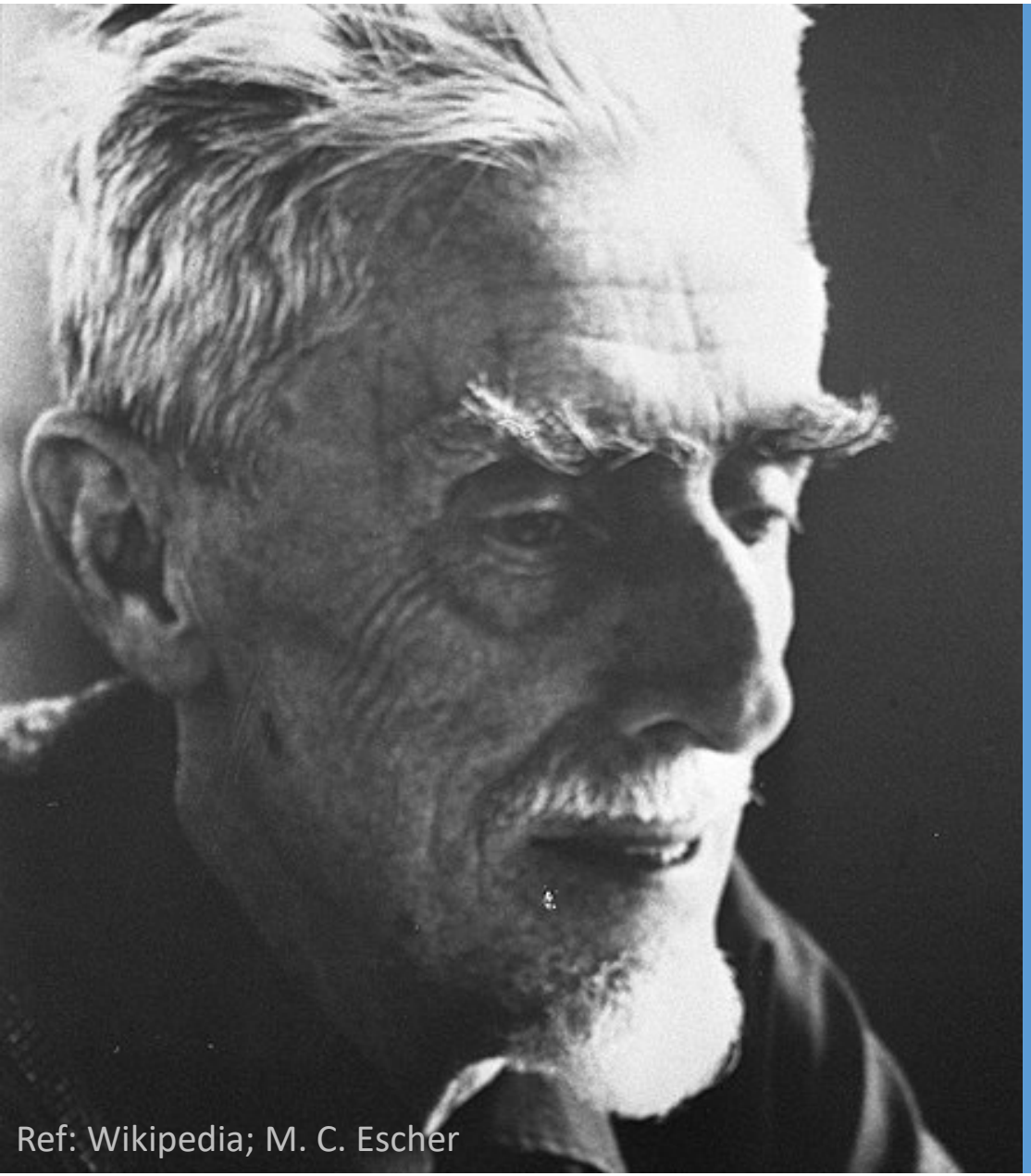
Marginal Problems

C-Y Hsieh, G N M Tabia, Y-C Yin, Y-C Liang, arXiv:2202.03523

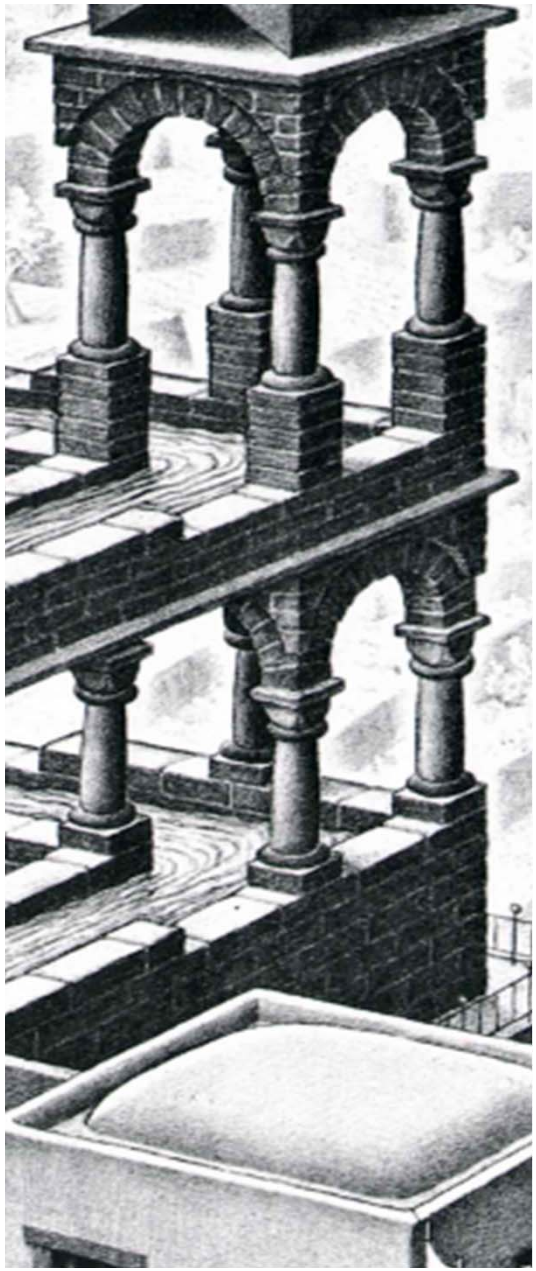
What's that?

Marginal Problems



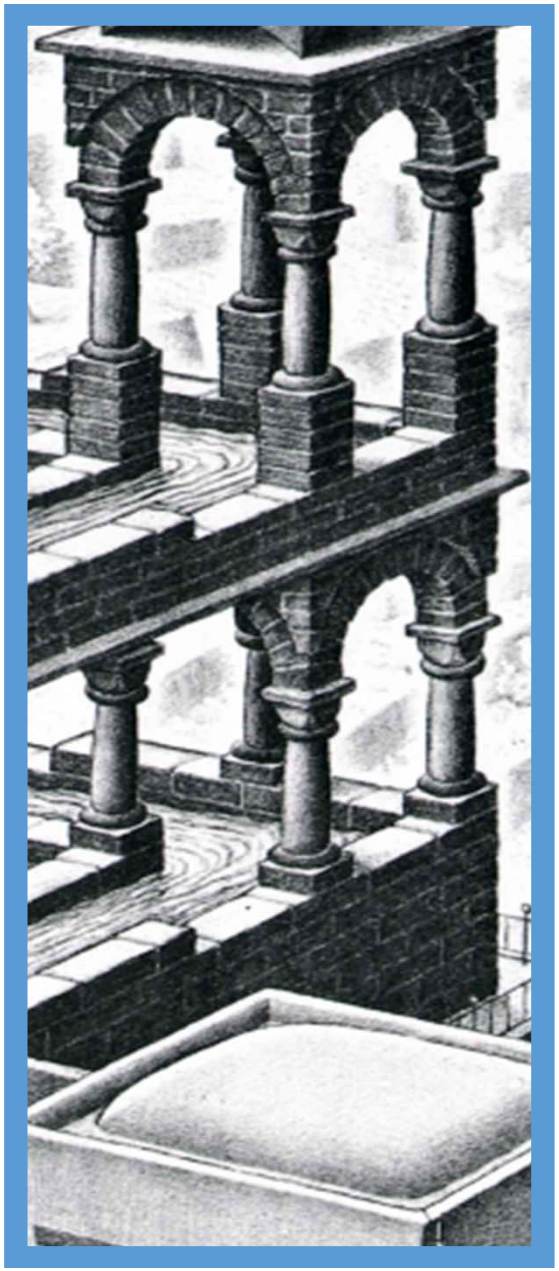
Ref: Wikipedia; M. C. Escher

Arts
by
M. C. Escher



M. C. Escher,
Cascada (Waterfall) (1961)

Image from
https://historia-arte.com/



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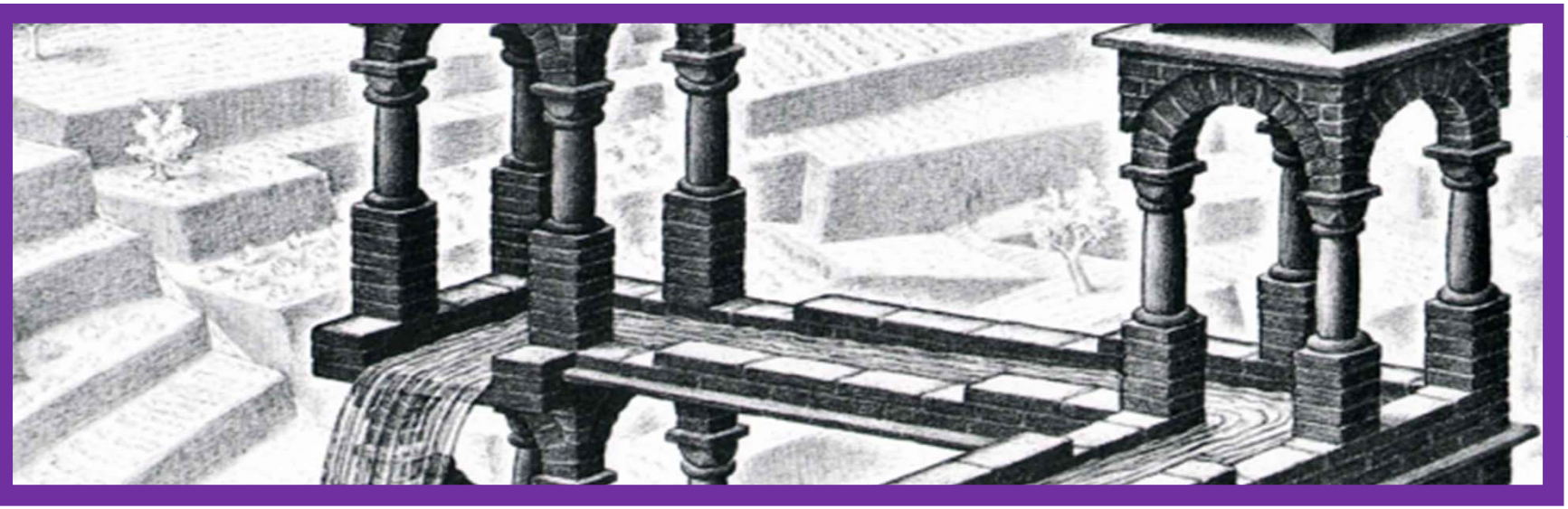

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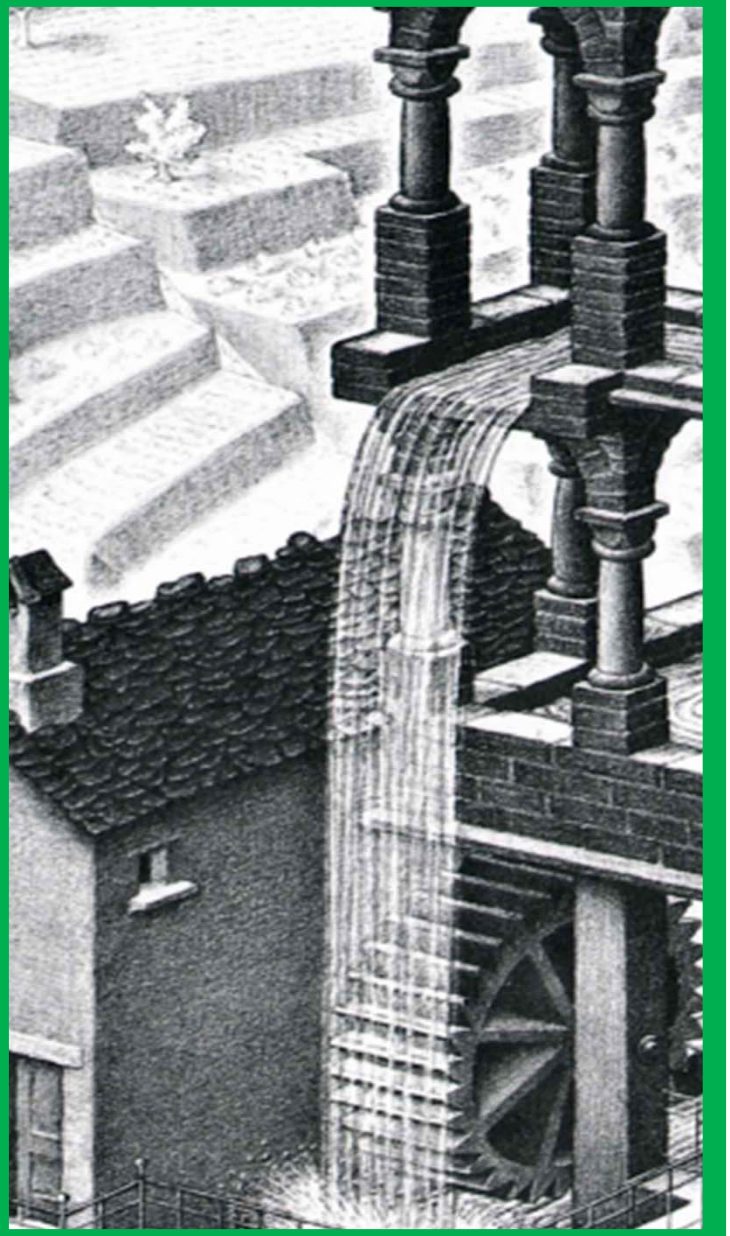
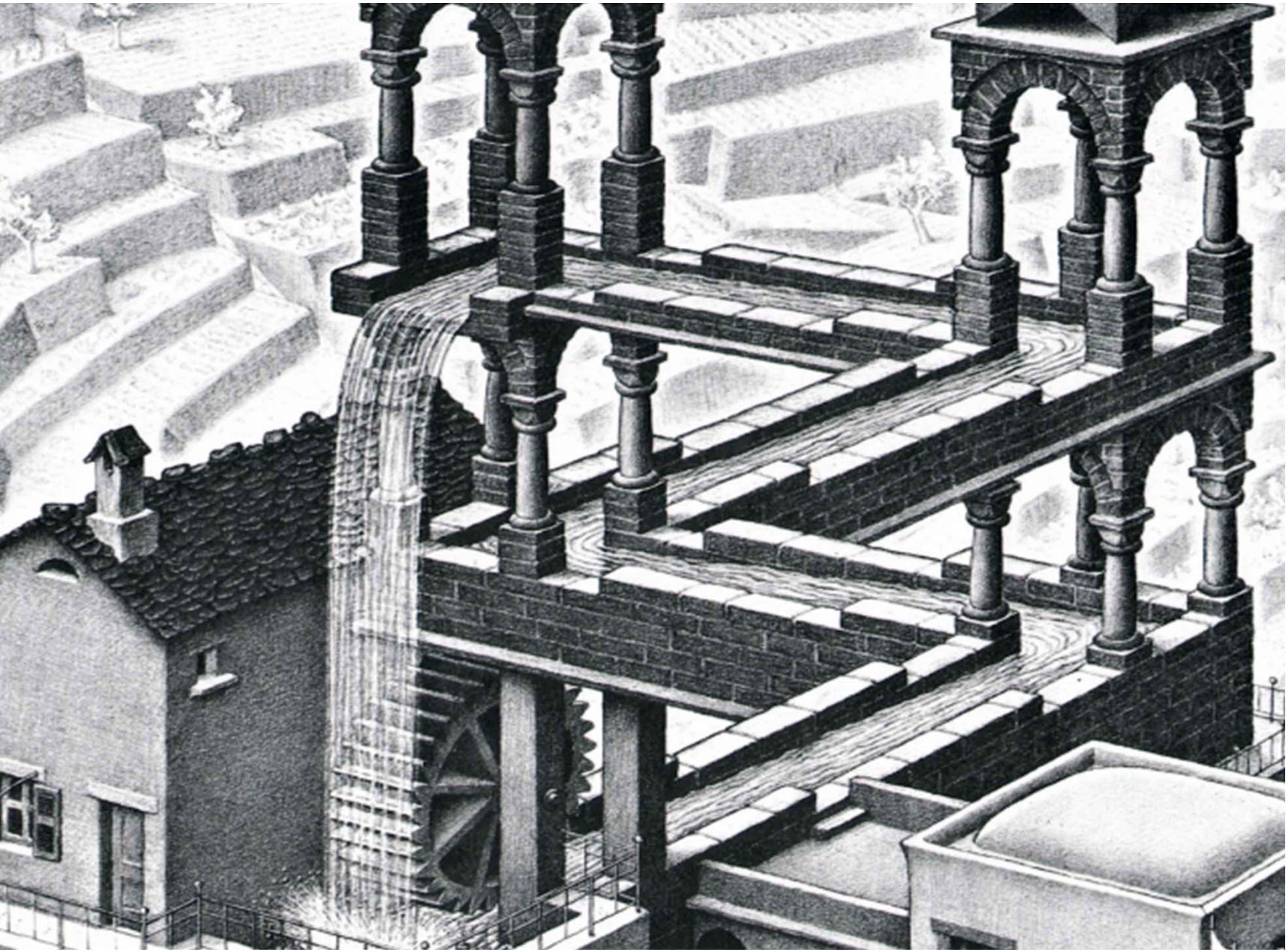


M. C. Escher,
Cascada (Waterfall) (1961)

Image from
https://historia-arte.com/

Can they coexist?



M. C. Escher,
Cascada (Waterfall) (1961)

Image from
https://historia-arte.com/

Locally compatible

Globally incompatible

Locally compatible

Globally incompatible

Marginal Problem

Can they coexist?

Marginal Problem

Probability Distr.

Can they coexist?

Classical Marginal Problem

Quantum States

Can they coexist?

State Marginal Problem

Quantum States

Can they coexist?

J Tura, R Augusiak, A B Sainz, T Vértesi, M Lewenstein, A Acín, Science **344**, 1256 (2014).

M Navascués, F Baccari, A Acín, Quantum **5**, 589 (2021).

State Marginal Problem

Nonlocality Detection

Entanglement theory

Static Marginal Problem

Dynamical Marginal Problem

Quantum dynamics

Can they coexist?

Channel Marginal Problem

Q Channel

R esource

Marginal Problems

C-Y Hsieh, M Lostaglio, A Acín, *Phys. Rev. Research* **4**, 013249 (2022)

Q Channel

R esource

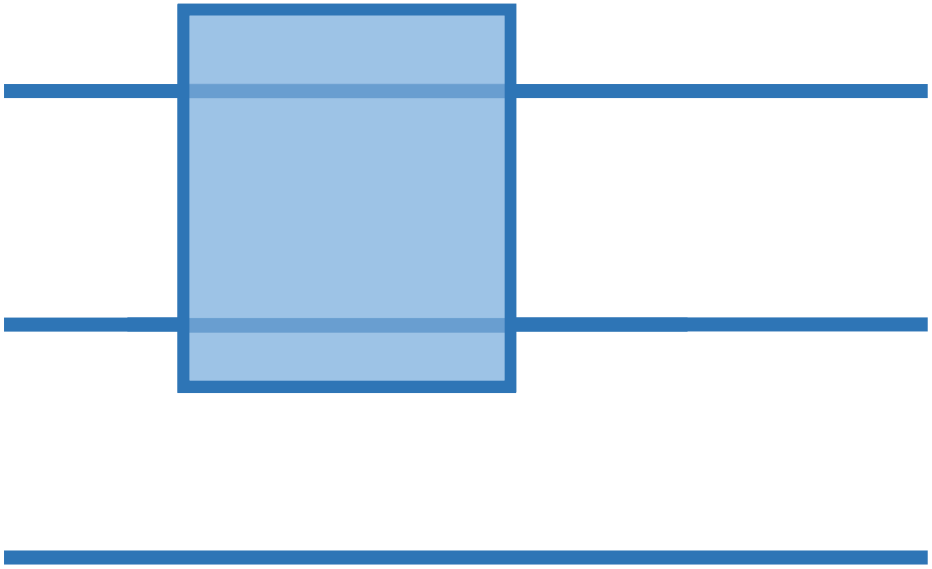
Marginal Problems

A Dynamical Generalization of State Marginal Problems

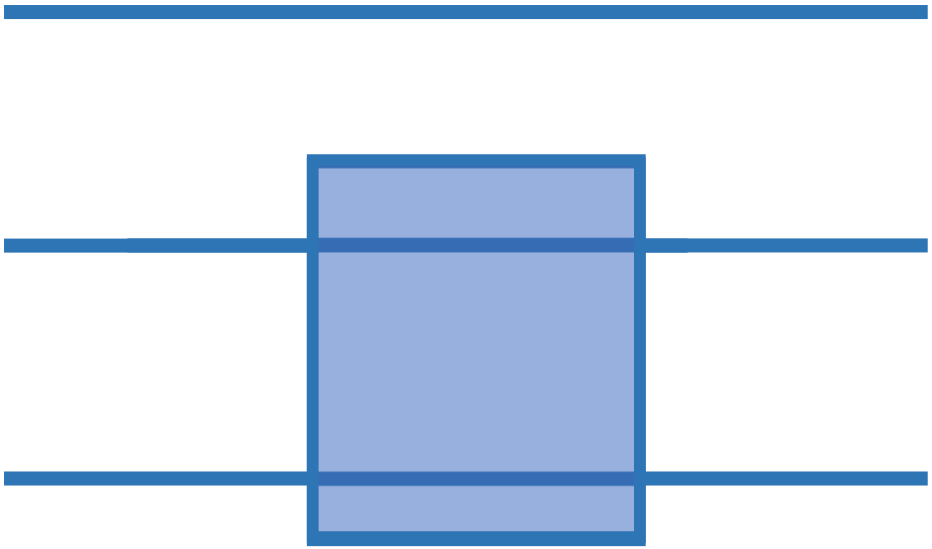
Given some local quantum dynamics, can they coexist?

Given some local quantum dynamics, can they coexist?

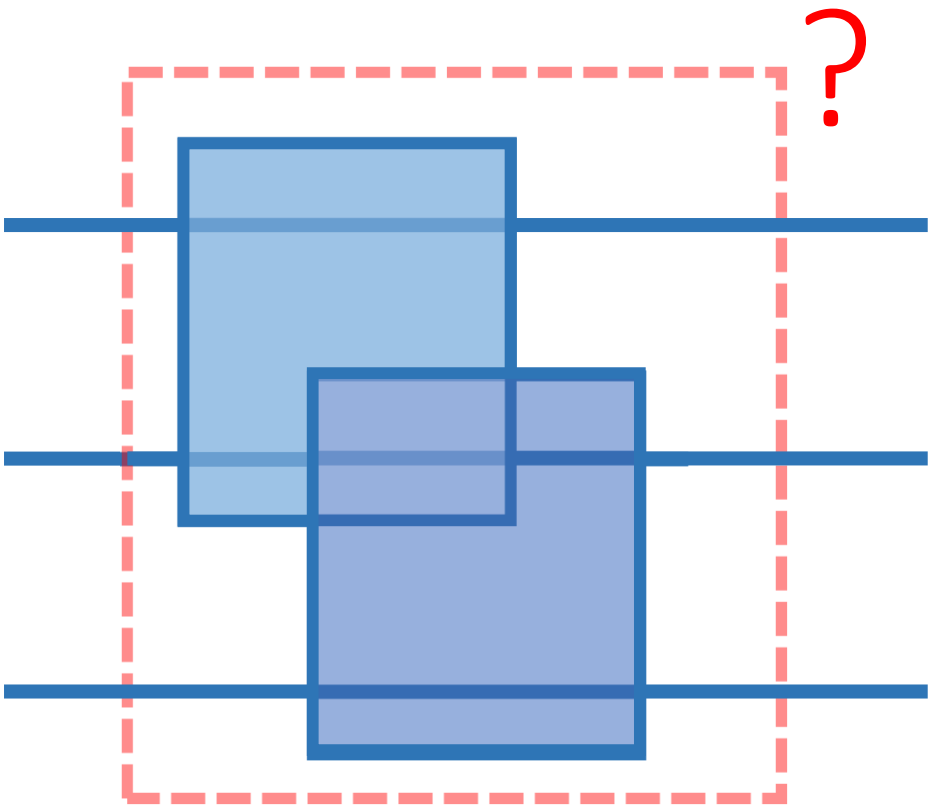
Given some local quantum dynamics, can they coexist?



Given some local quantum dynamics, can they coexist?



Given some local quantum dynamics, can they coexist?

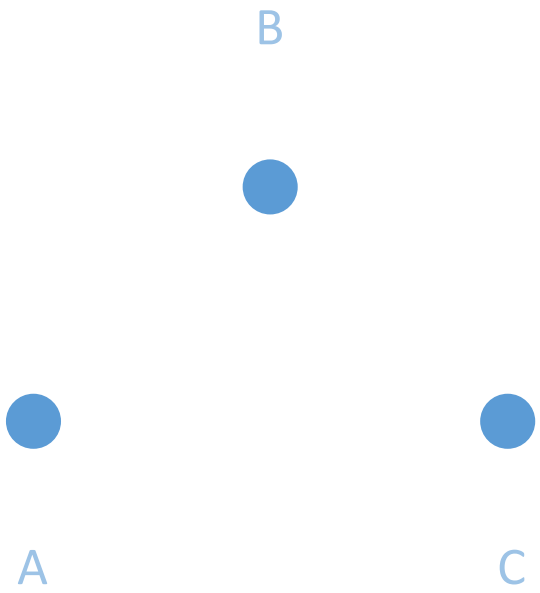


Given some local quantum dynamics, can they coexist?

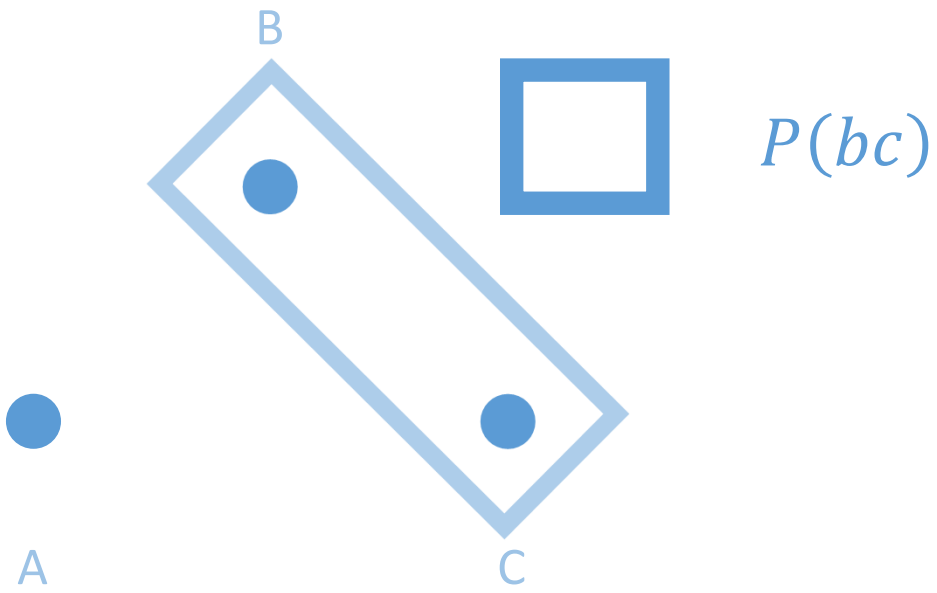
But...

Why Quantum Marginal Problem?

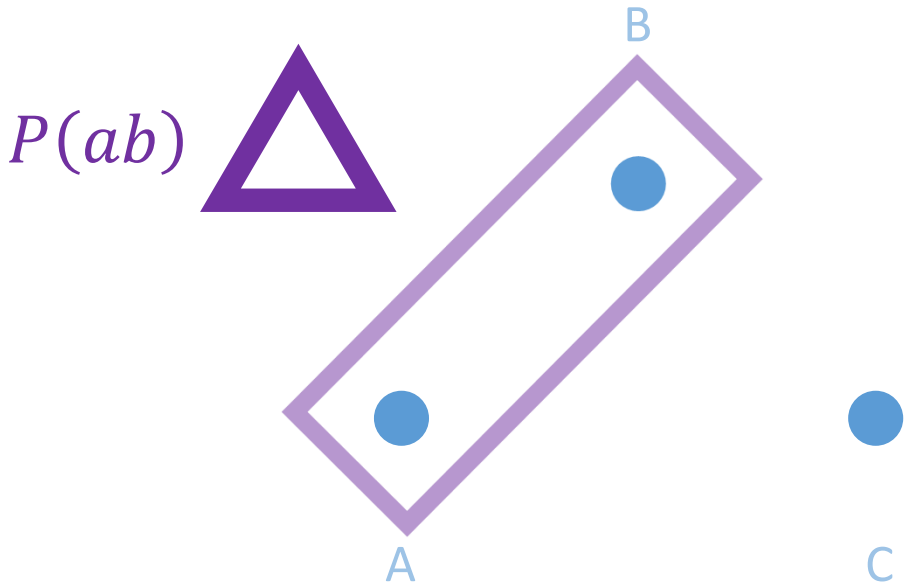
Why Dynamical Marginal Problem?



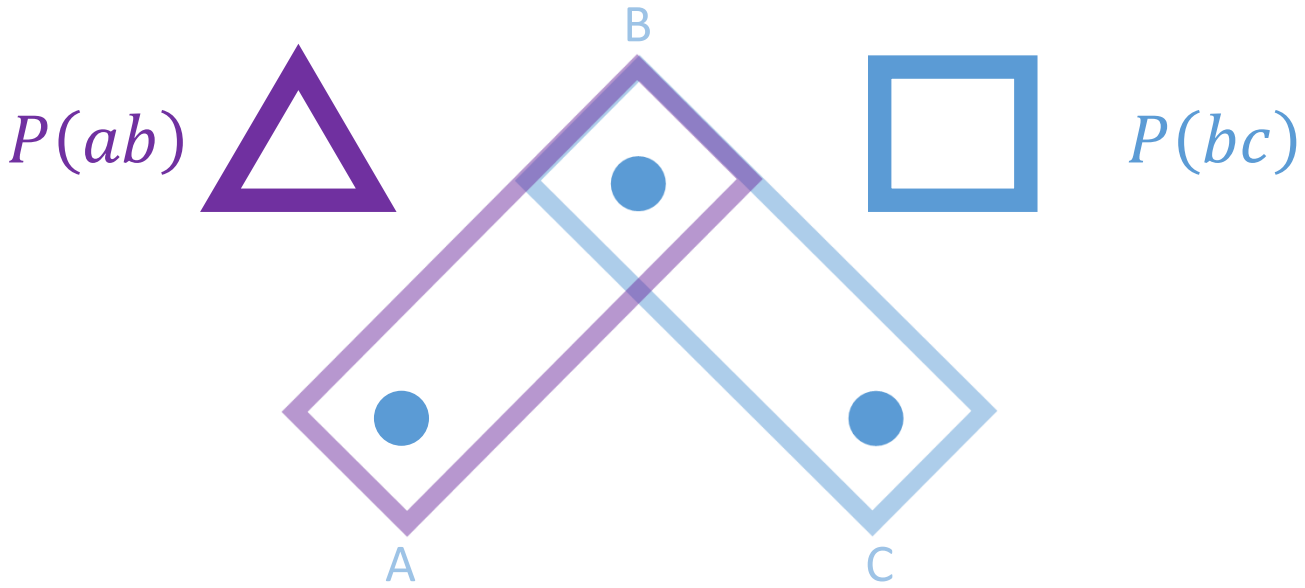
Classical



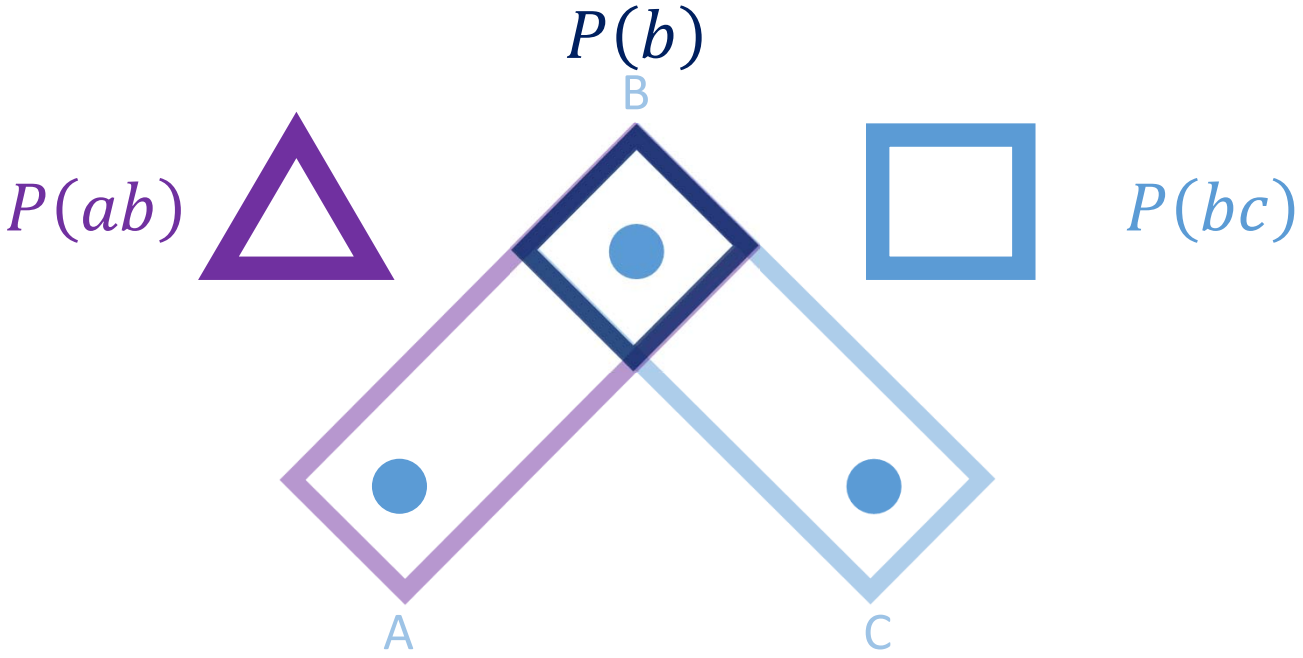
Classical



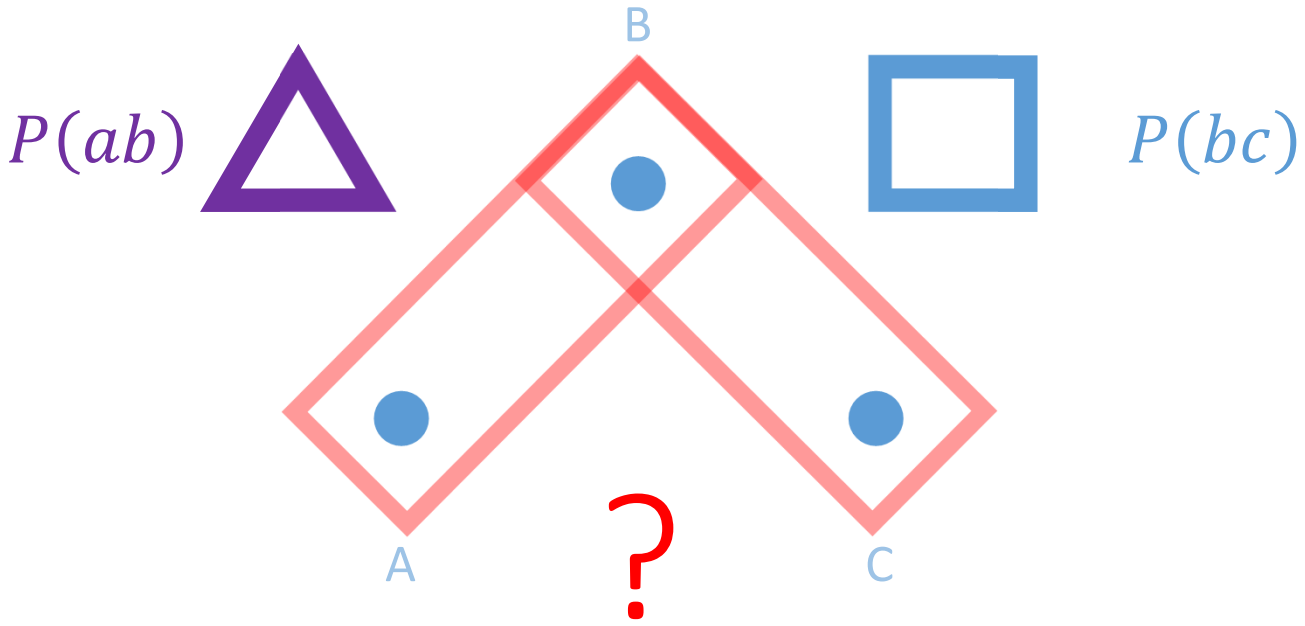
Classical



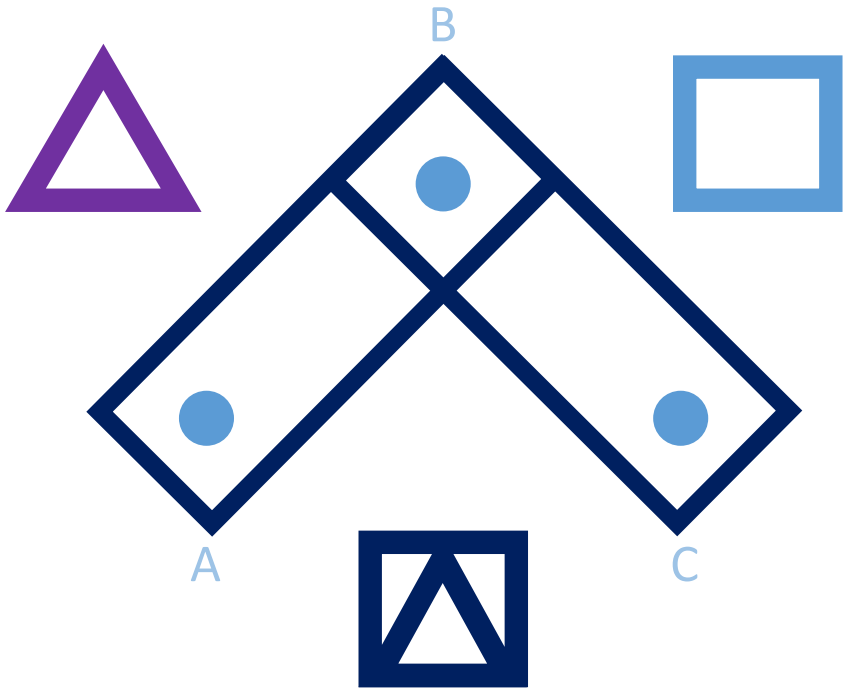
Classical



Classical

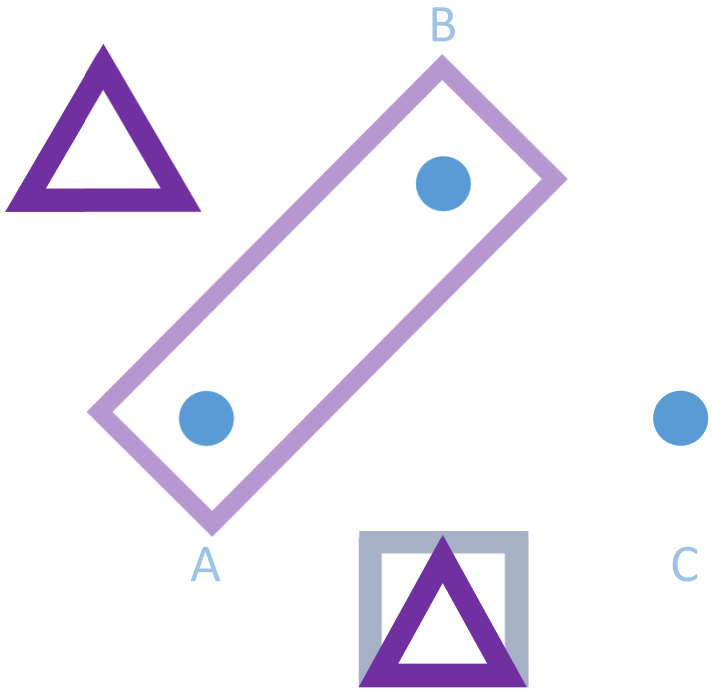


Classical



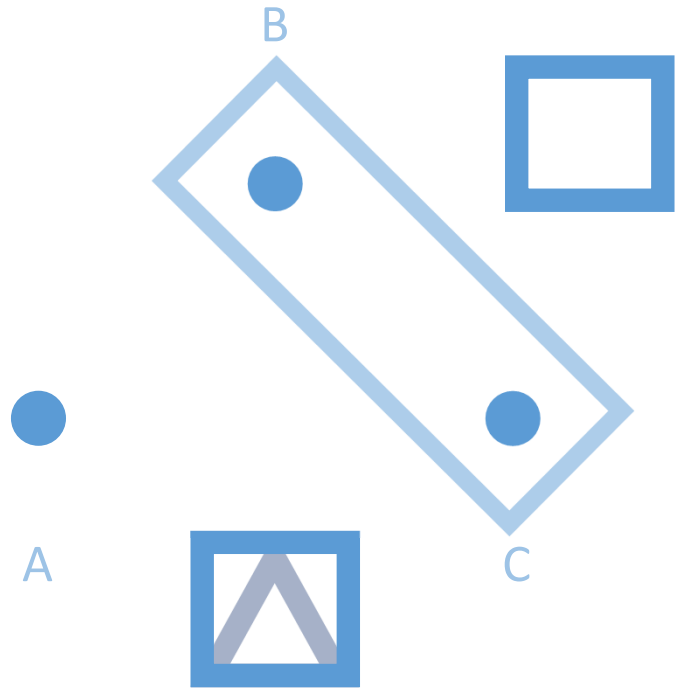
$$P(abc) = \frac{P(ab)P(bc)}{P(b)}$$

Classical



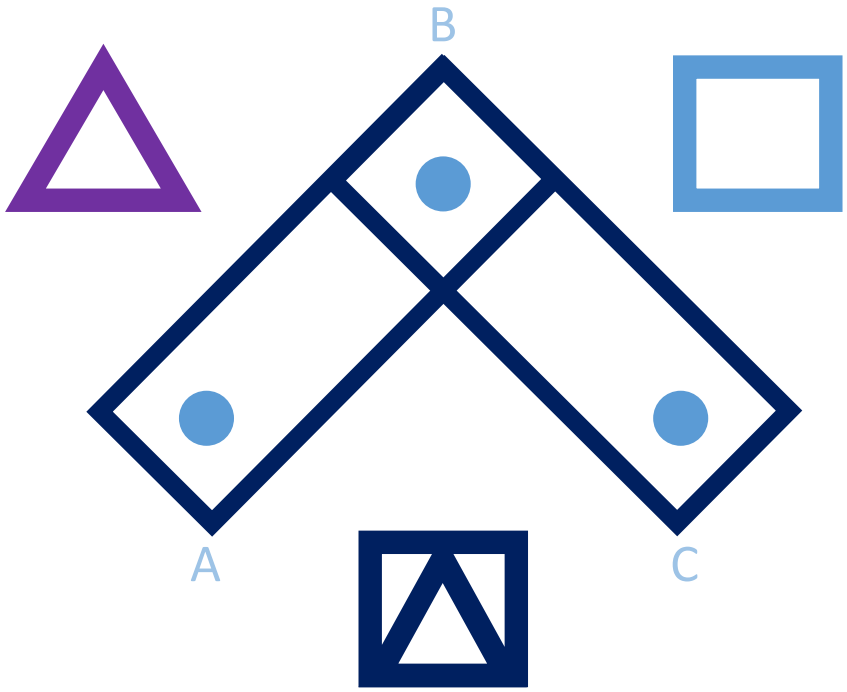
$$\sum_c P(abc) = \sum_c \frac{P(ab)P(bc)}{P(b)} = P(ab)$$

Classical



$$\sum_a P(abc) = \sum_a \frac{P(ab)P(bc)}{P(b)} = P(bc)$$

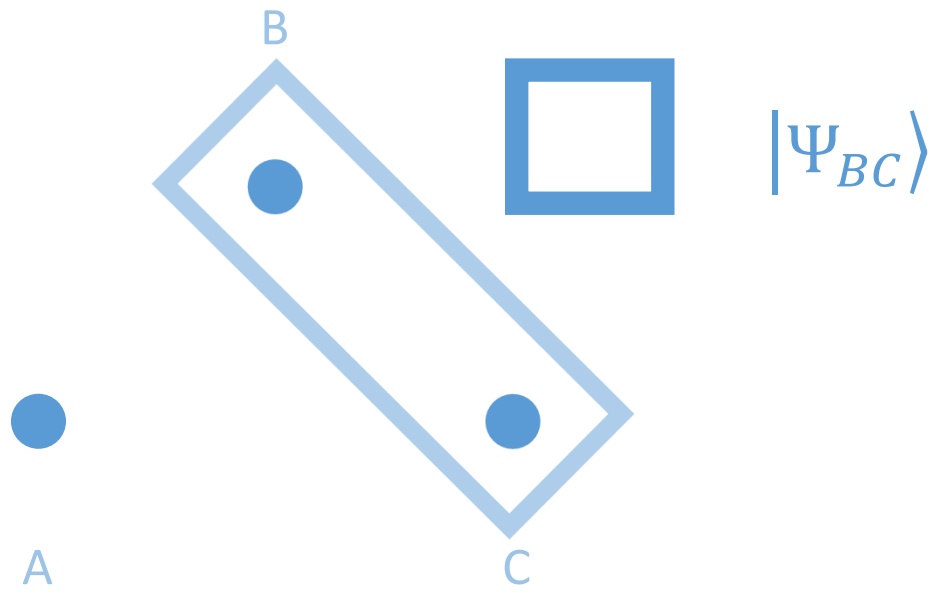
Classical



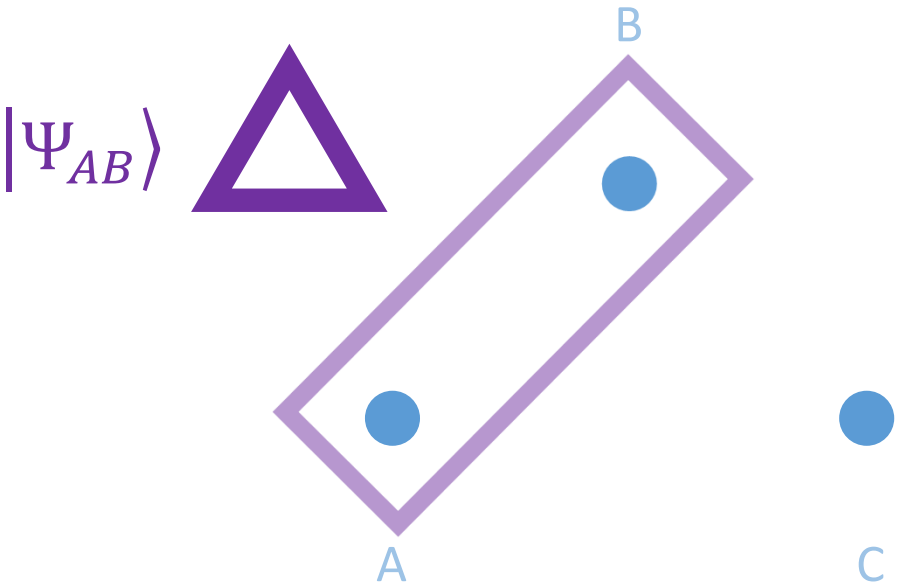
In AB/BC setting, both static and dynamical classical marginal problems are trivial

Locally compatible ⟹ Globally compatible

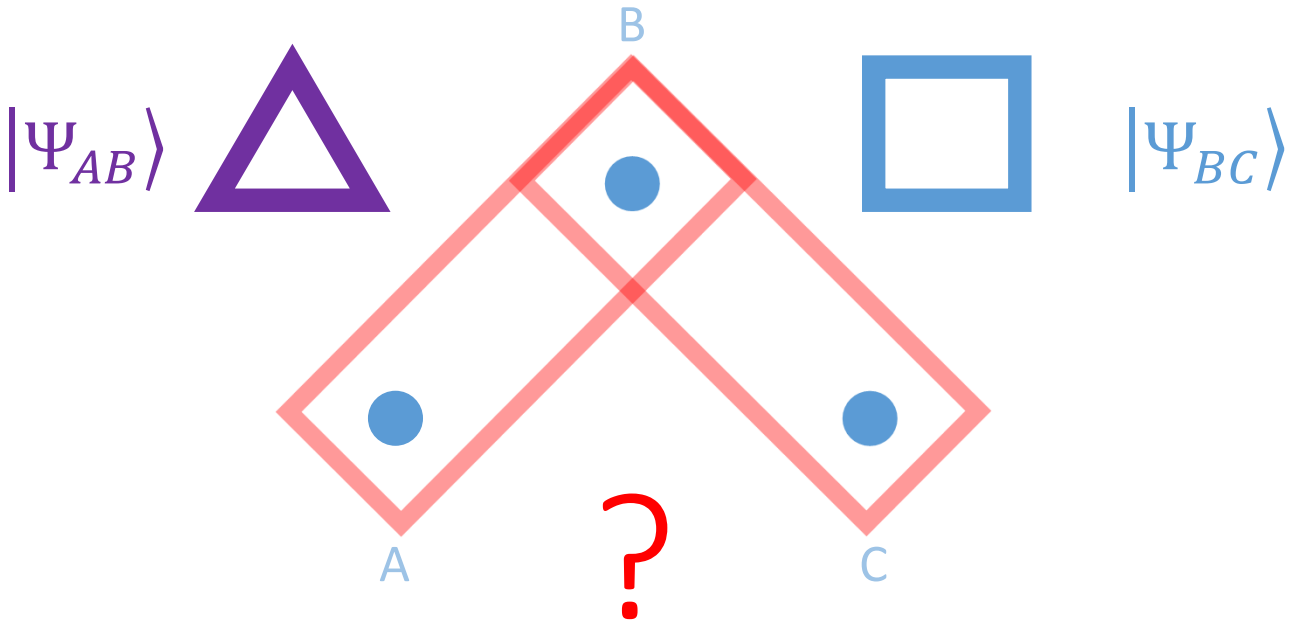
Classical



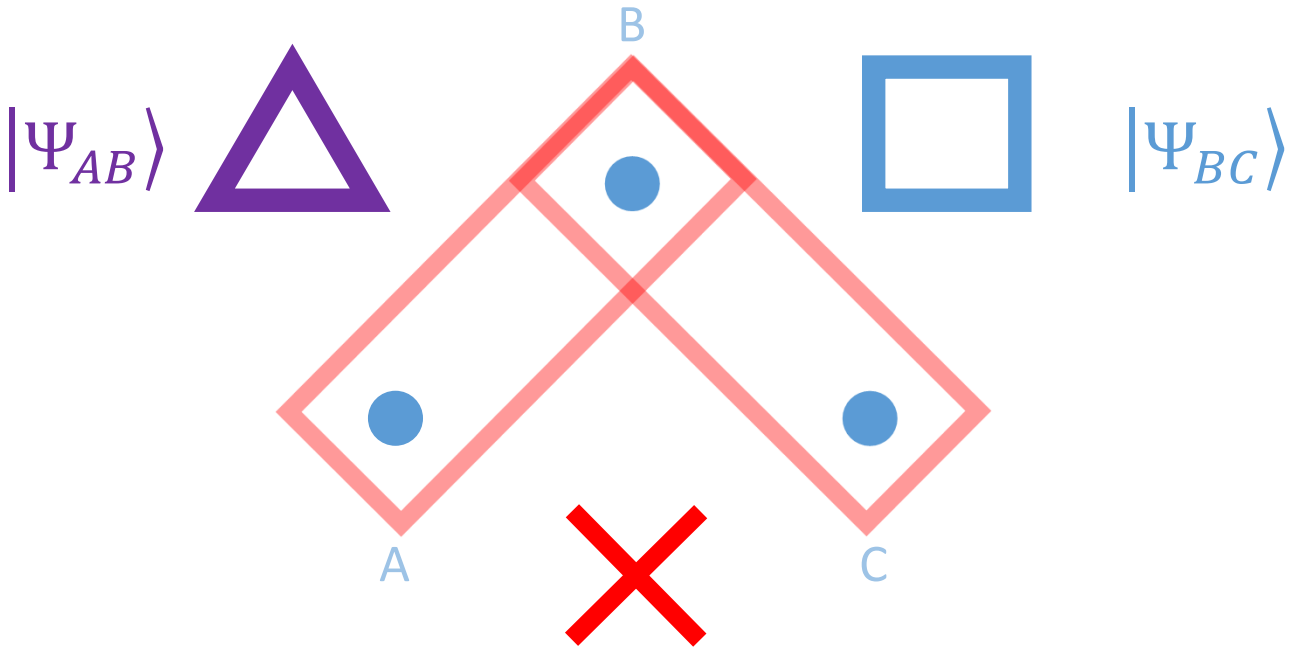
Quantum States



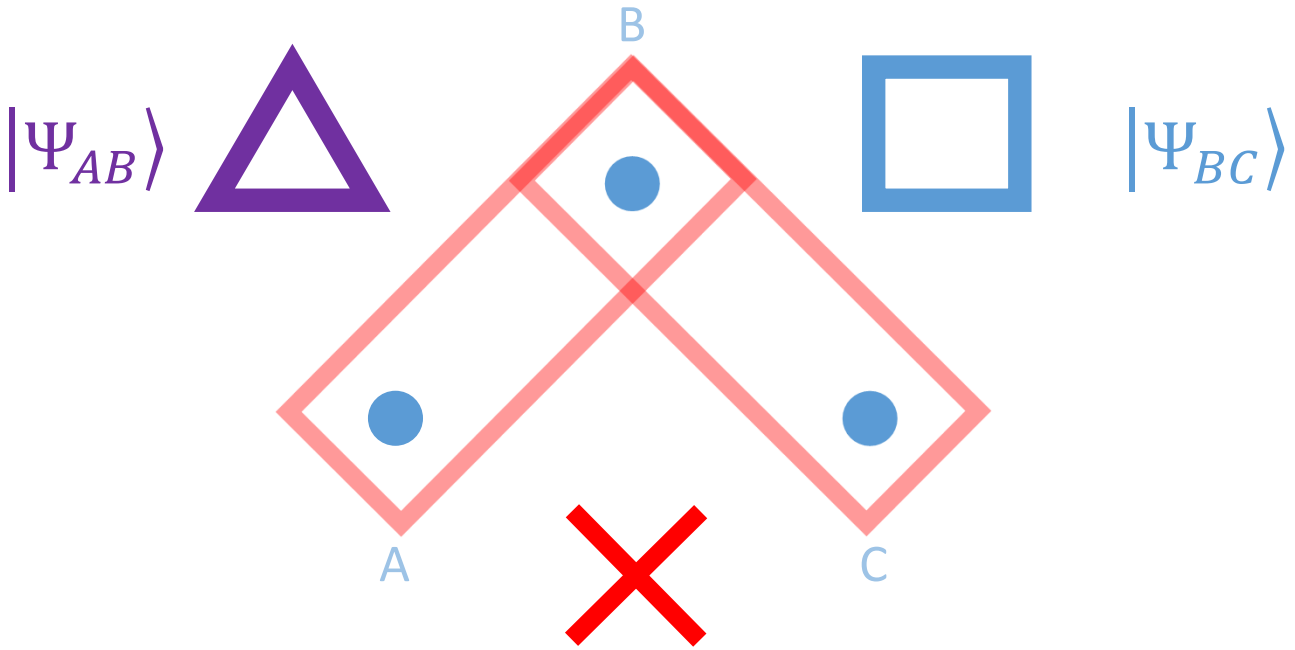
Quantum States



Quantum States

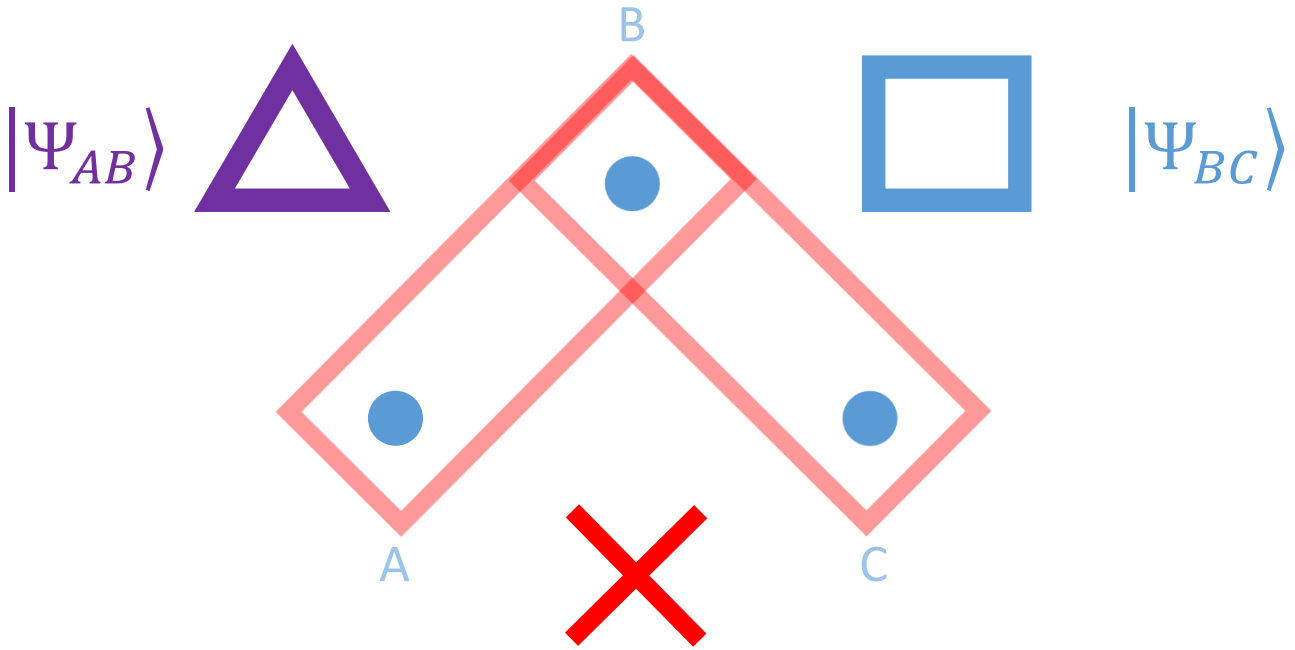


Entanglement is monogamous!



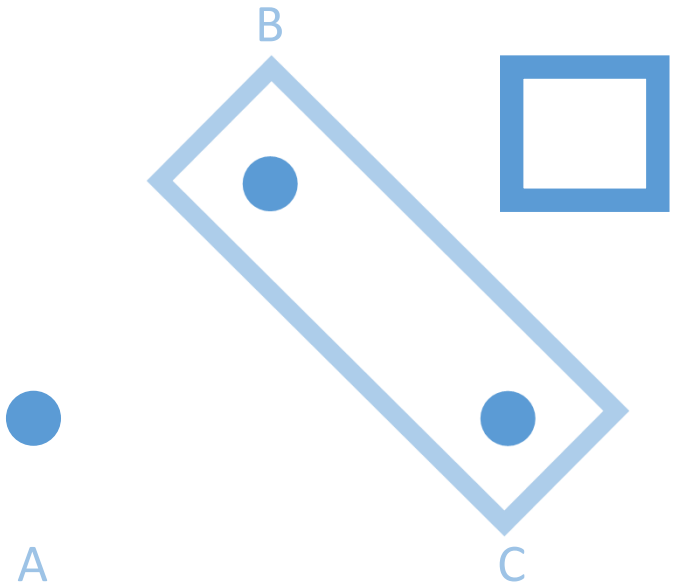
AB/BC state marginal problem is nontrivial: e.g. entanglement monogamy

Why Quantum Marginal Problem?



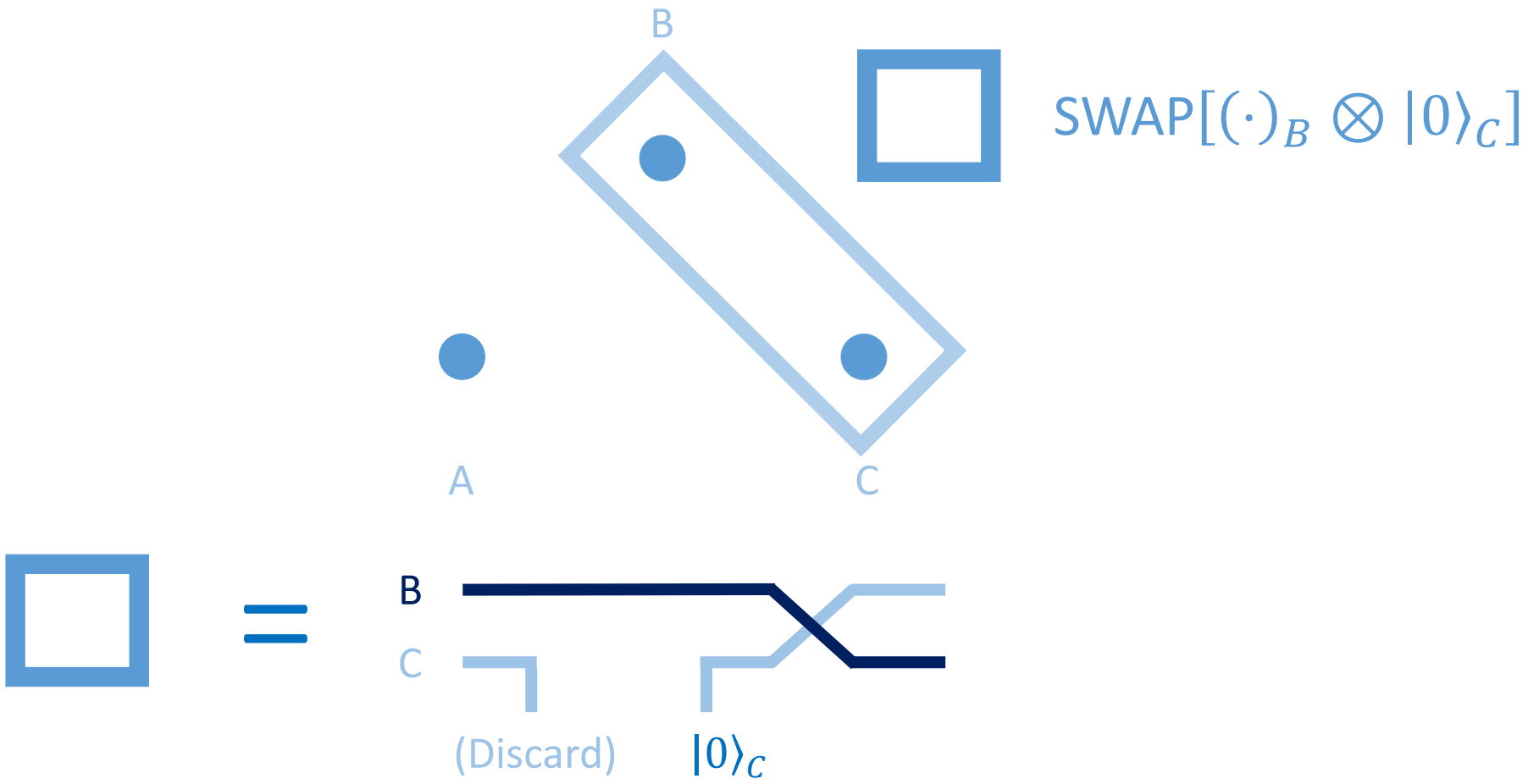
AB/BC state marginal problem is nontrivial: e.g. entanglement monogamy

Marginal Problem: Quantum > Classical



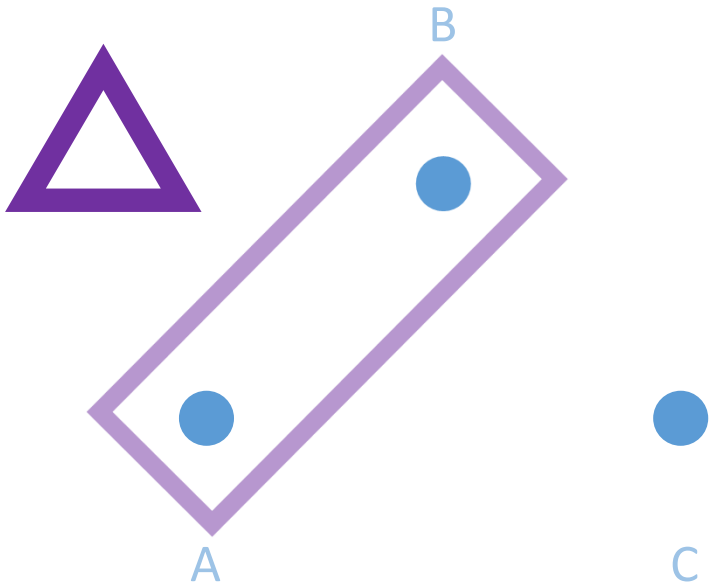
$\mathrm{SWAP}[(\cdot)_B \otimes |0\rangle_C]$

Quantum Dynamics



Quantum Dynamics

$\mathrm{SWAP}[(\cdot)_B \otimes |0\rangle_A]$



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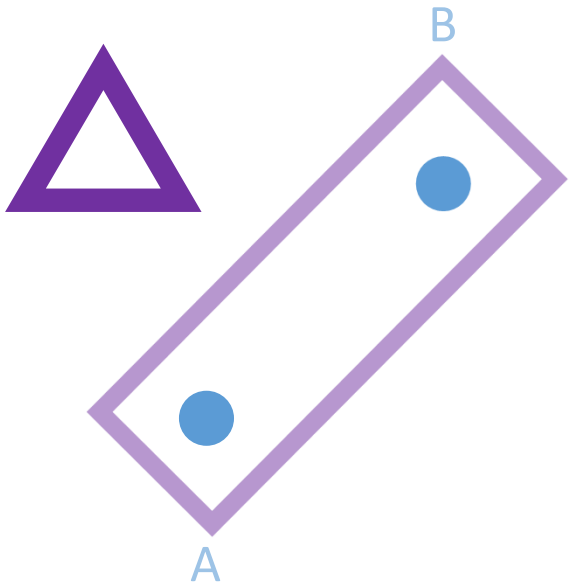
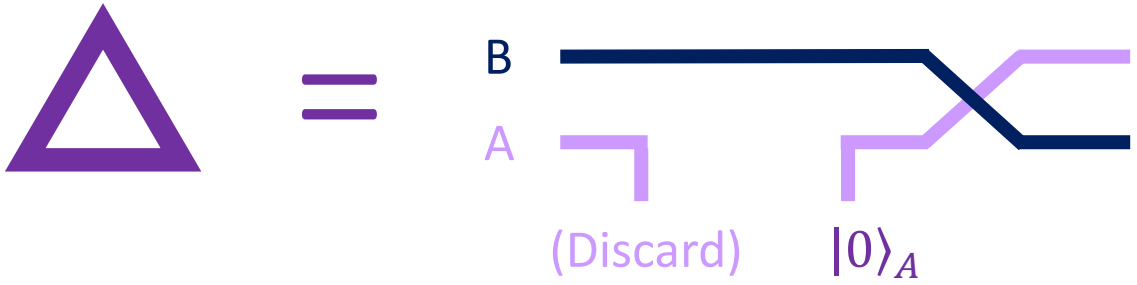
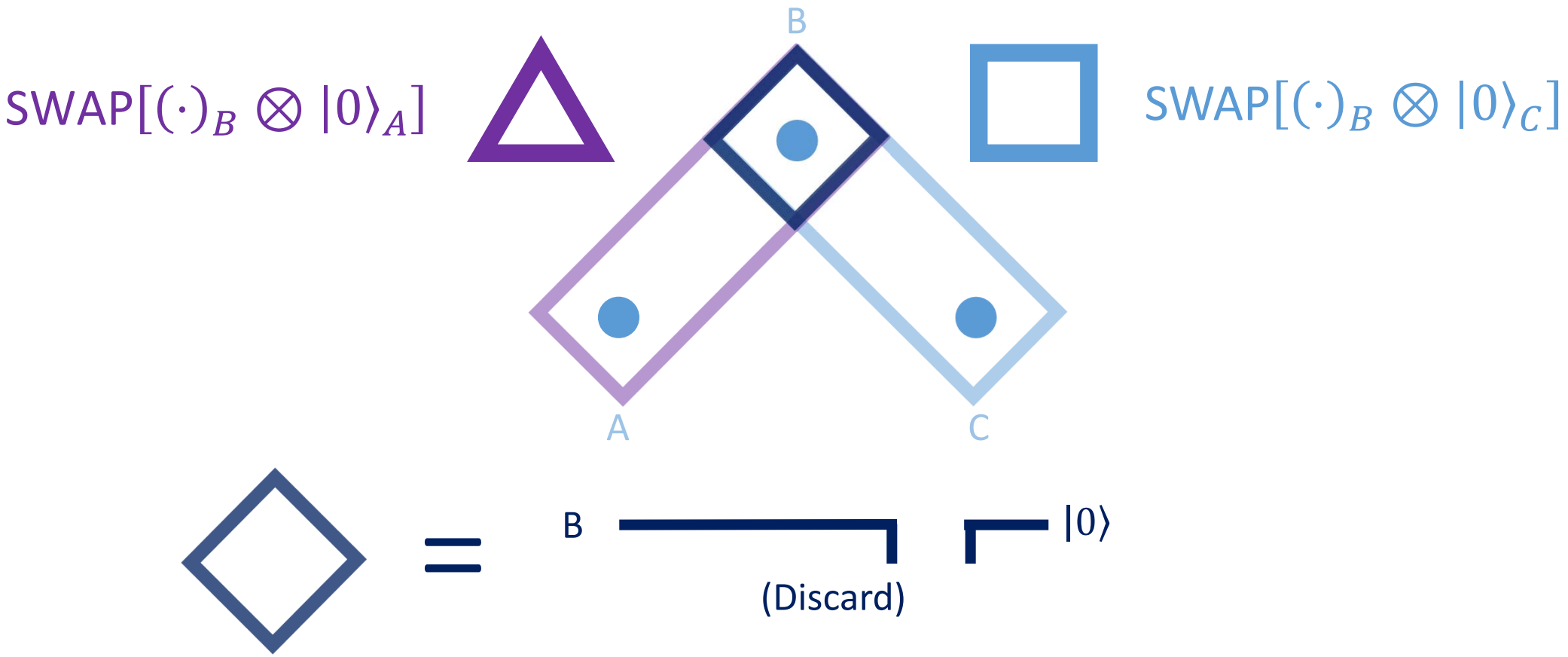


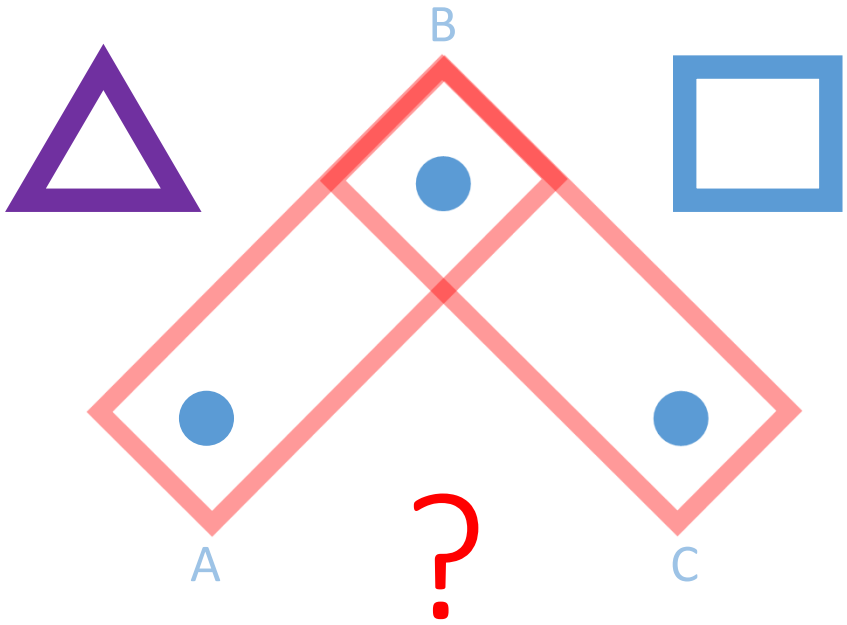


Quantum Dynamics


$\mathrm{SWAP}[(\cdot)_B \otimes |0\rangle_A]$
B
A
C
$\mathrm{SWAP}[(\cdot)_B \otimes |0\rangle_C]$
=
B
$|0\rangle$
(Discard)

Classical

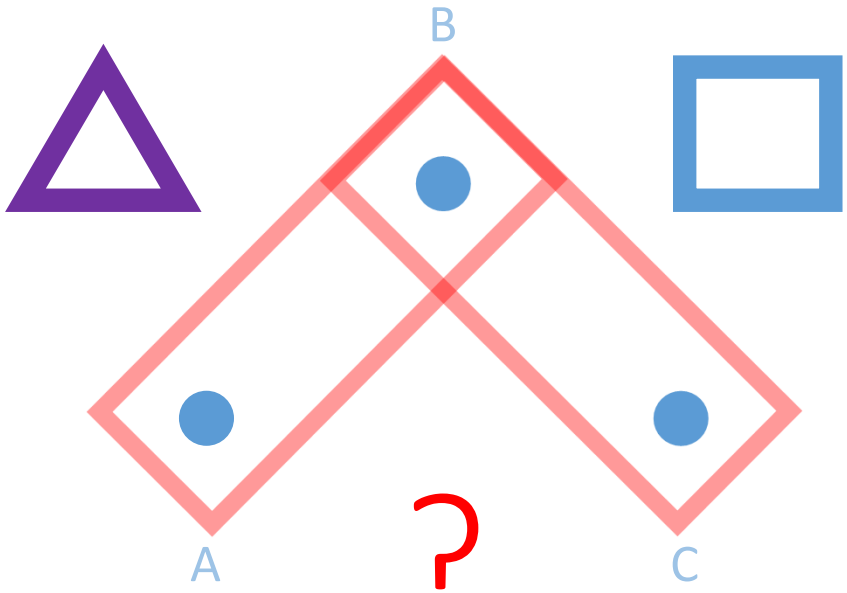
$\mathrm{SWAP}[(\cdot)_B \otimes |0\rangle_A]$



$\mathrm{SWAP}[(\cdot)_B \otimes |0\rangle_C]$

Quantum Dynamics

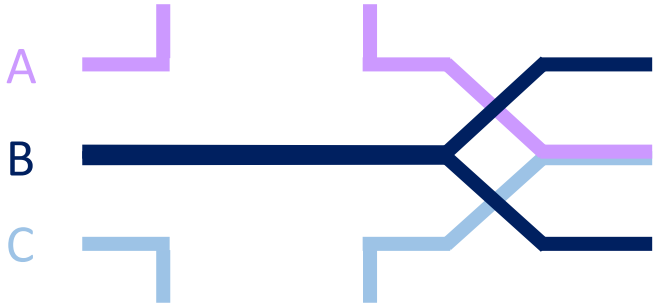
$\mathrm{SWAP}[(\cdot)_B \otimes |0\rangle_A]$



$\mathrm{SWAP}[(\cdot)_B \otimes |0\rangle_C]$

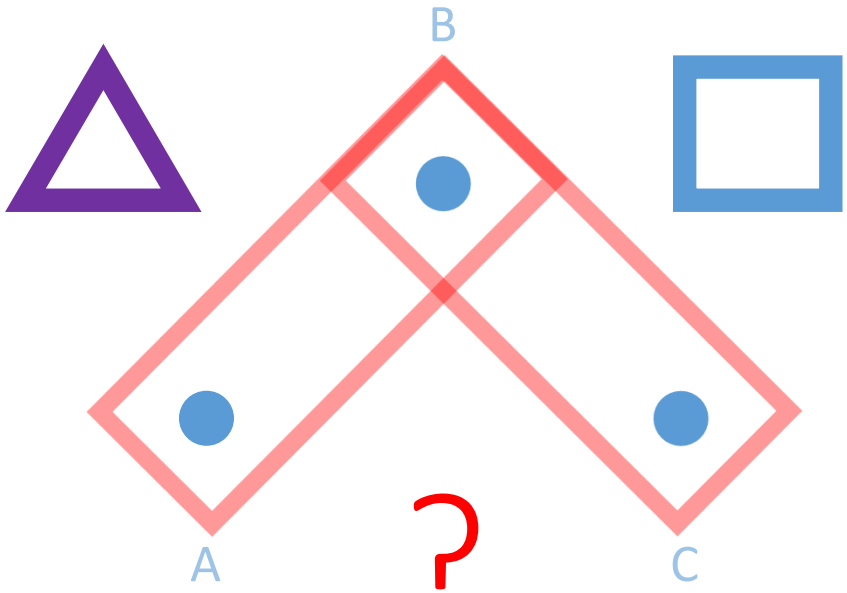
?

if so...



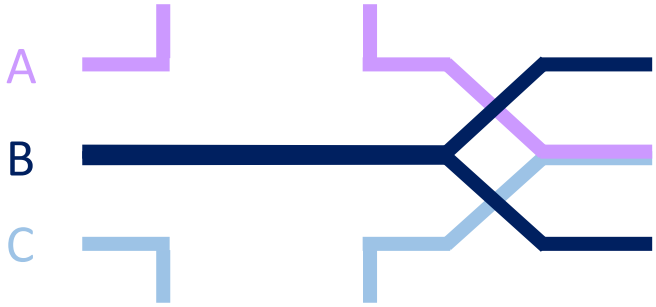
Quantum Dynamics

$\mathrm{SWAP}[(\cdot)_B \otimes |0\rangle_A]$



$\mathrm{SWAP}[(\cdot)_B \otimes |0\rangle_C]$

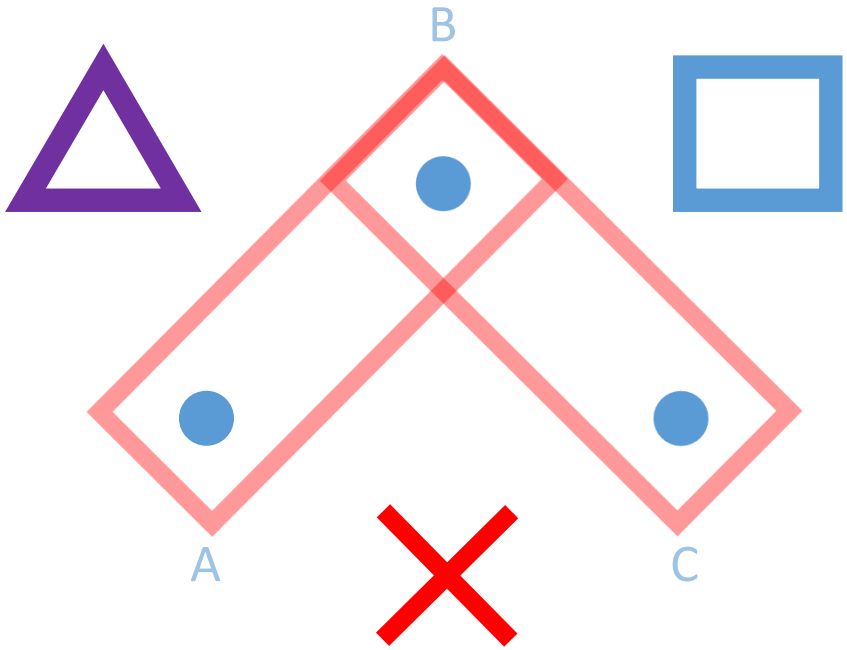
?



if so…

we can clone Q info!

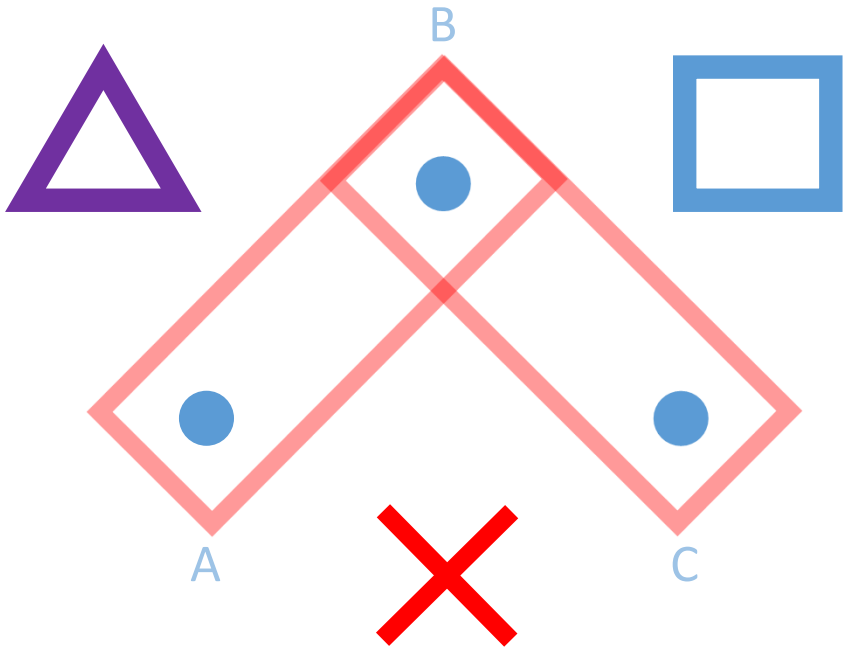
$\mathrm{SWAP}[(\cdot)_B \otimes |0\rangle_A]$



$\mathrm{SWAP}[(\cdot)_B \otimes |0\rangle_C]$

Quantum information cannot be cloned!

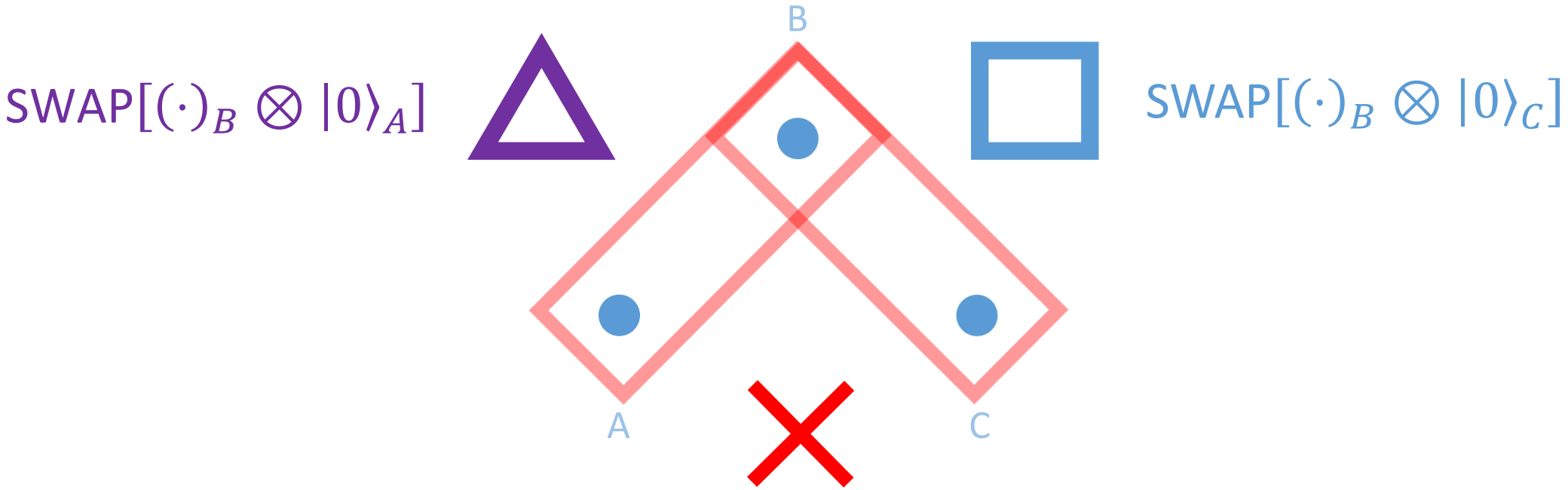
$\mathrm{SWAP}[(\cdot)_B \otimes |0\rangle_A]$



$\mathrm{SWAP}[(\cdot)_B \otimes |0\rangle_C]$

AB/BC dynamical marginal problem is nontrivial: e.g. no-cloning theorem

Why Dynamical Marginal Problem?



AB/BC dynamical marginal problem is nontrivial: e.g. no-cloning theorem

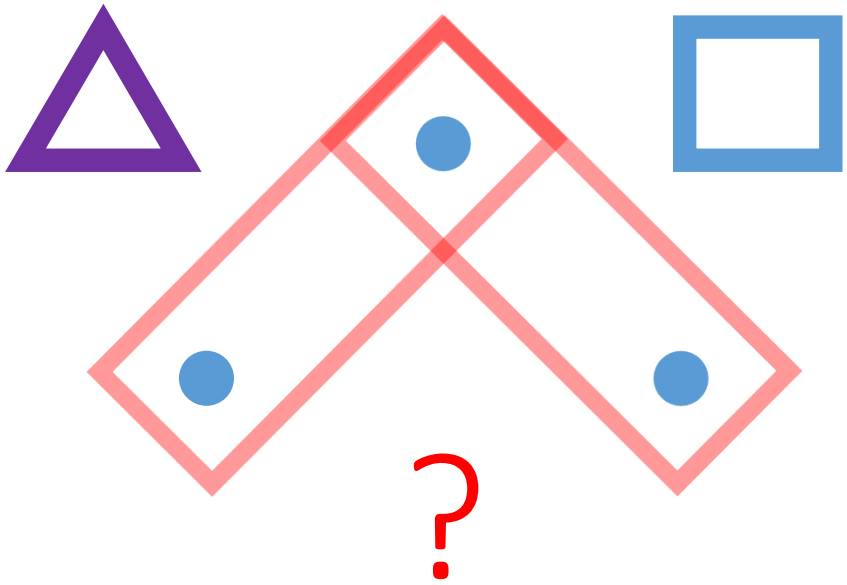
Marginal Problem: Dynamical ≠ Static

Qualitative Phenomenon

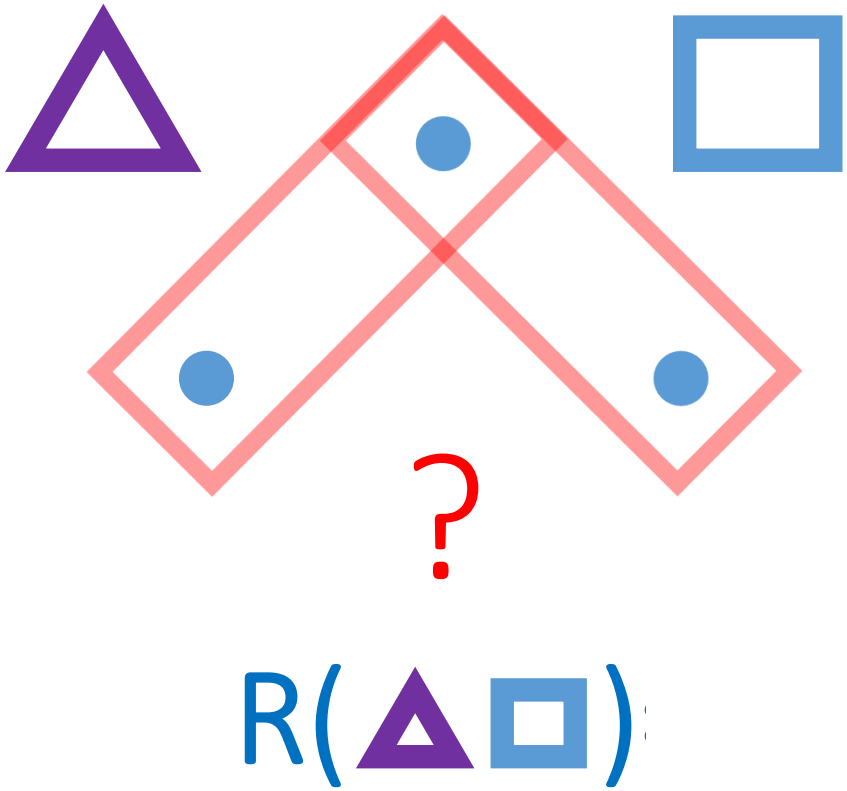
Quantitative Study

Quantitative Study

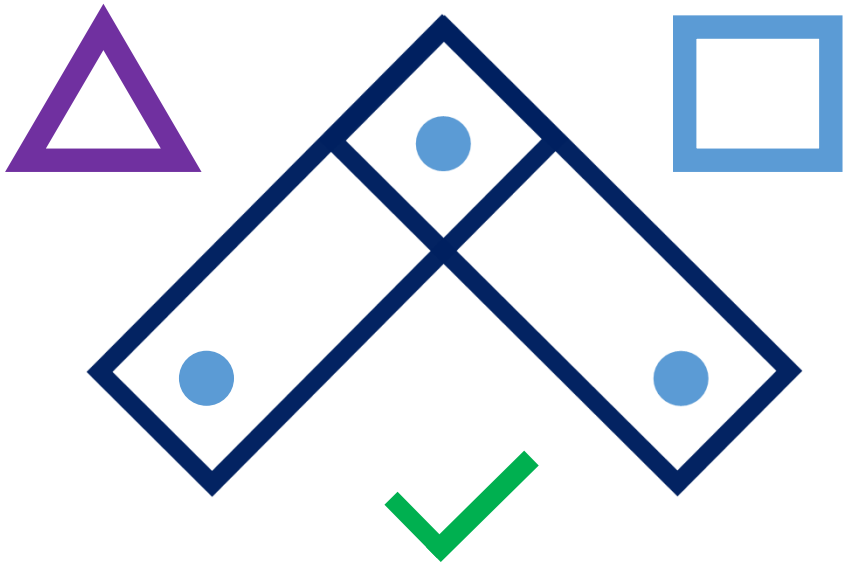
Given dynamics, how incompatible they are?



Quantum Dynamics

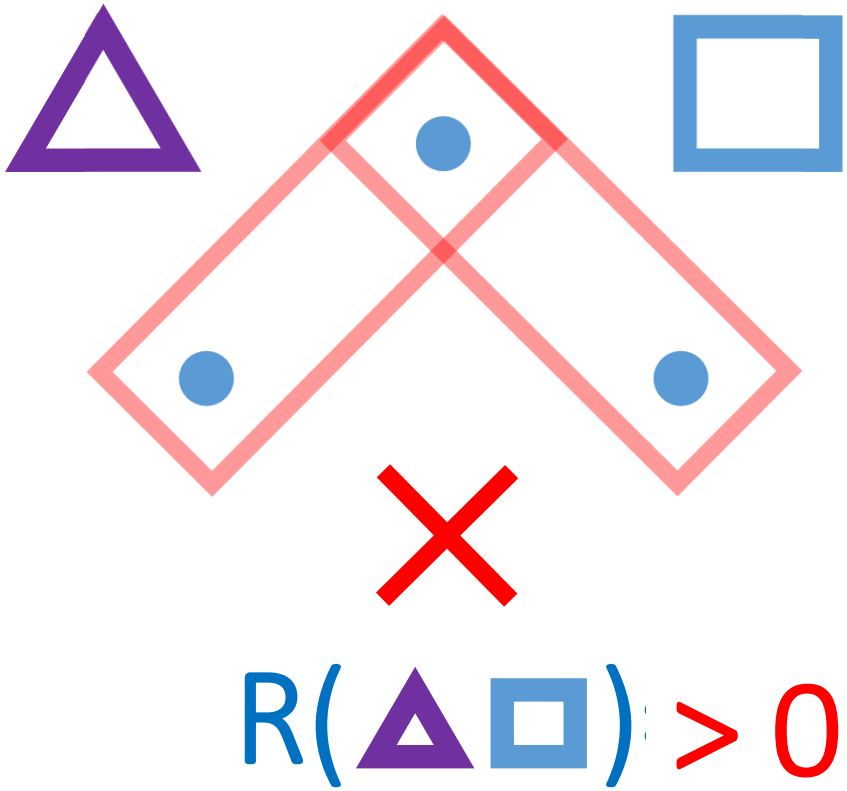


Quantum Dynamics

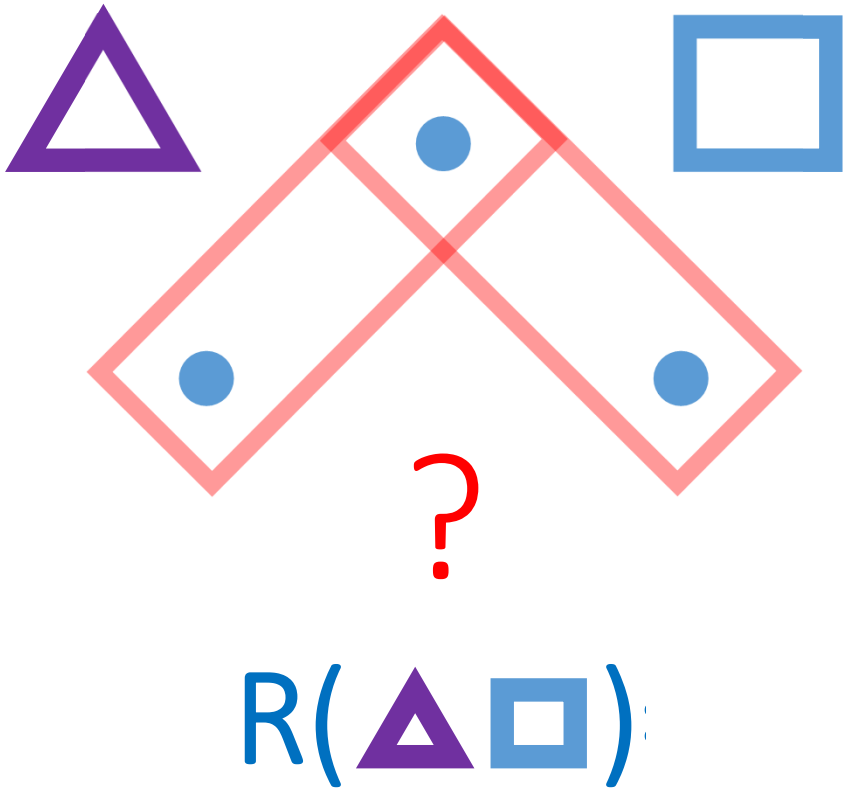


$R(\triangle\square) = 0$

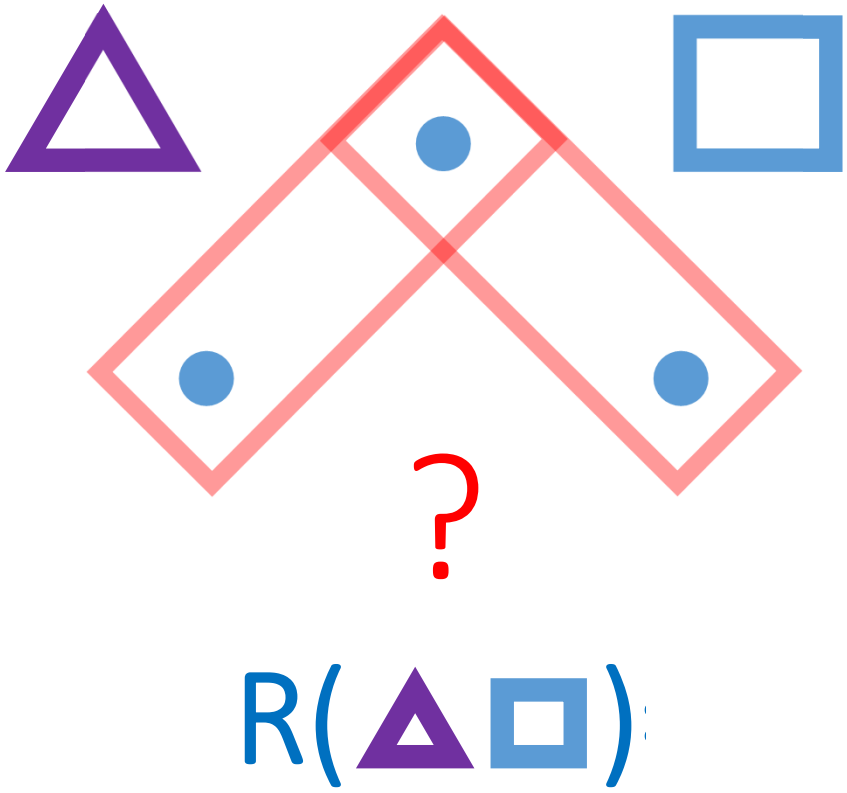
Quantum Dynamics



Quantum Dynamics



Quantifying Dynamical Incompatibility



Advantages in state discrimination tasks

Incompatibility is useful

Can compatibility also be useful?

Can compatibility certify a resource?

Q Channel

R esource

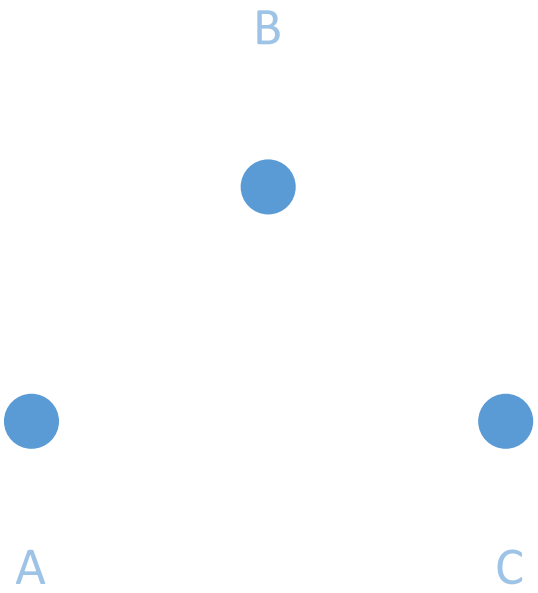
Marginal Problems

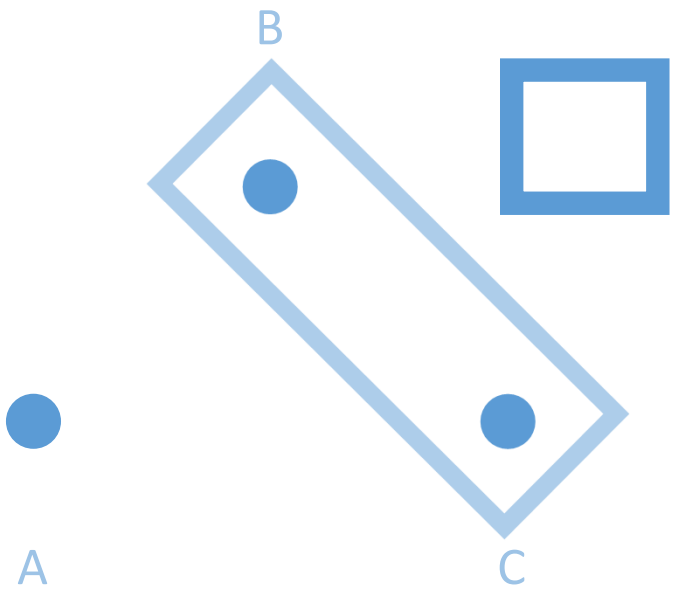
C-Y Hsieh, G N M Tabia, Y-C Yin, Y-C Liang, arXiv:2202.03523

Example

Transitivity of Q Resources

G N M Tabia, K-S Chen, C-Y Hsieh, Y-C Yin, Y-C Liang, *npj Quantum Inf.* **8**, 98 (2022)


B
A
C


B
A
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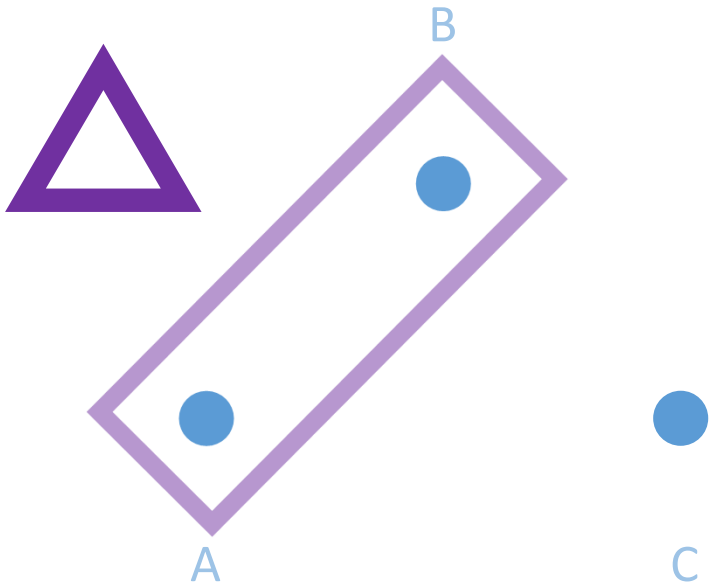
B
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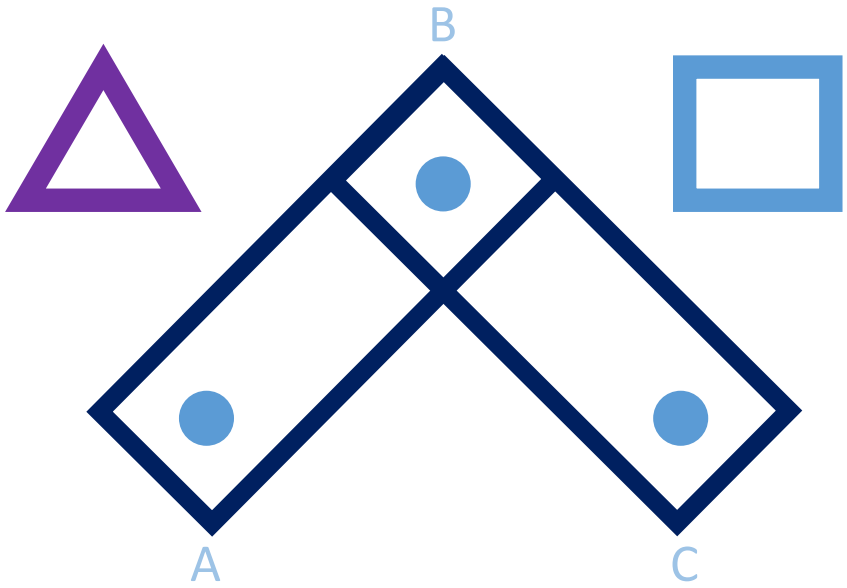
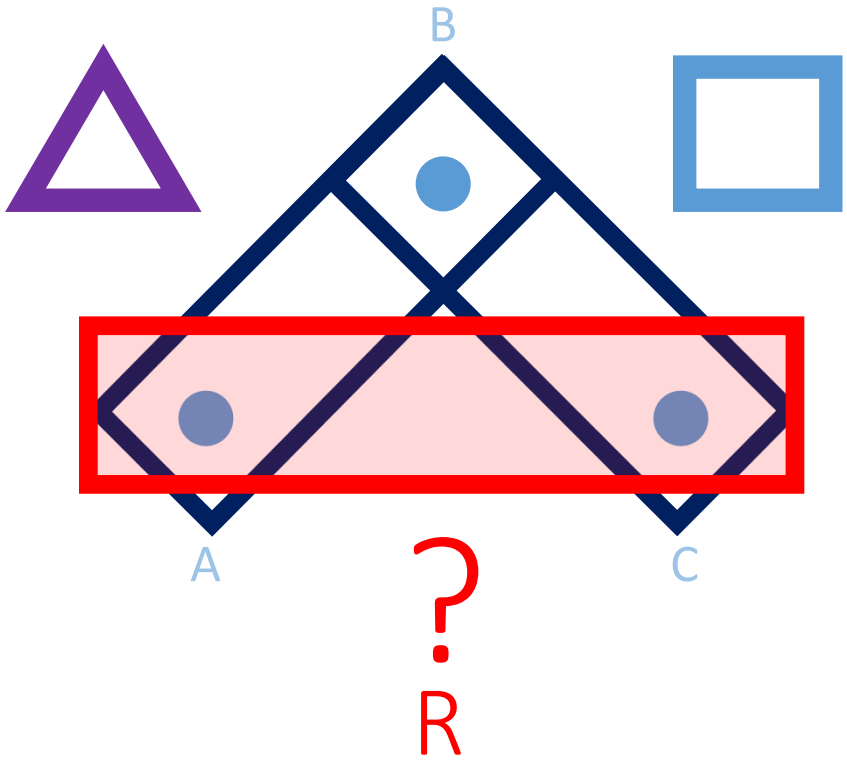
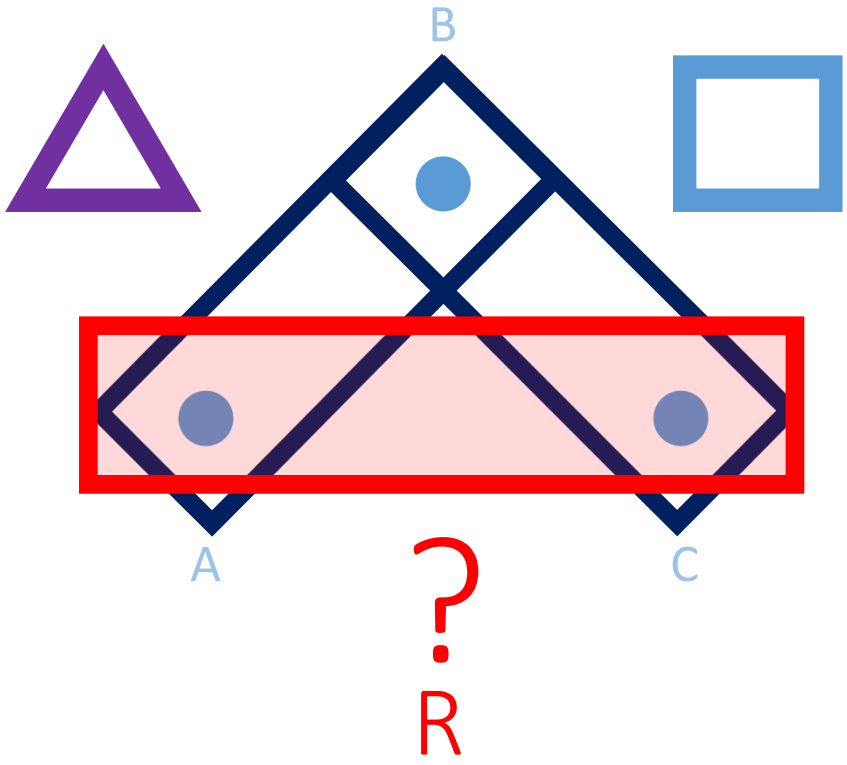
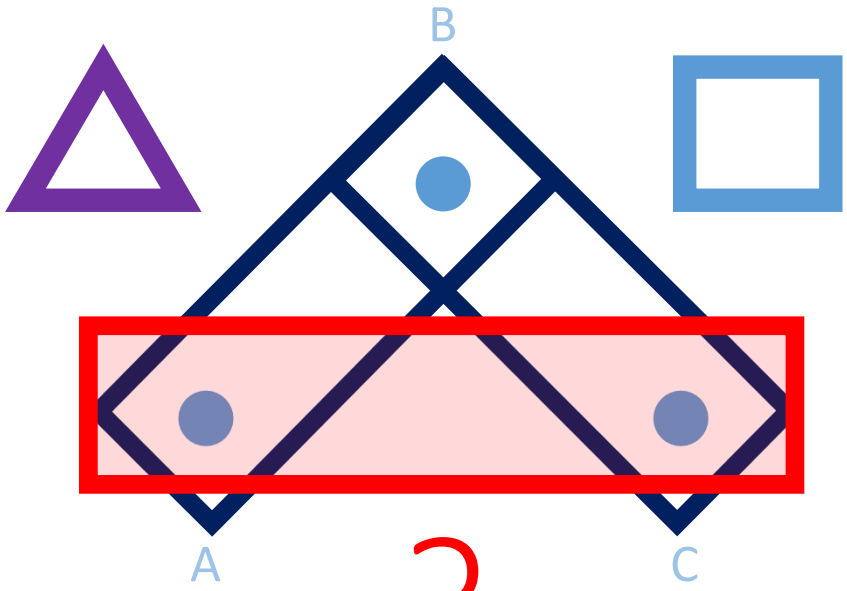

B
A
C

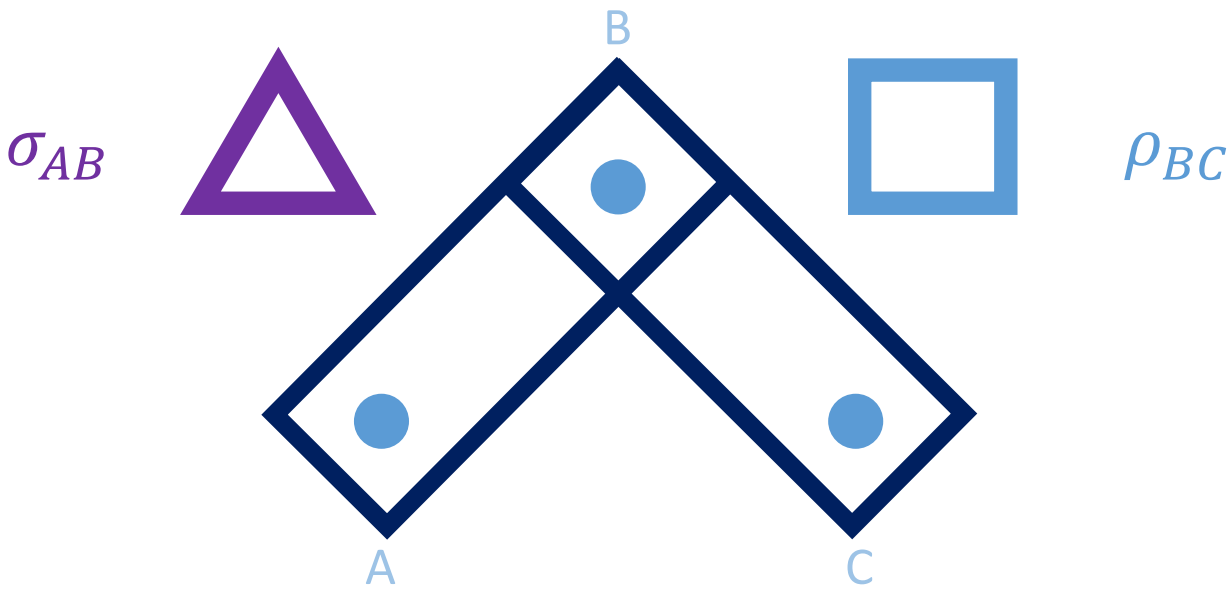

B
A
C
?
R

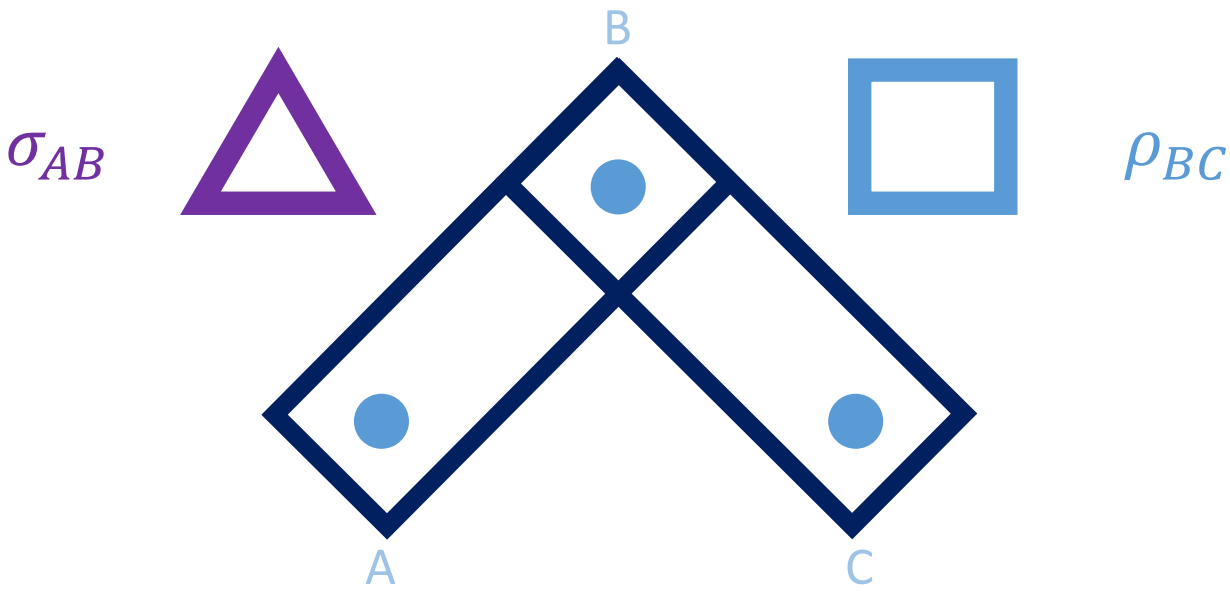


Given states in AB & BC, can they guarantee that AC marginal state is a resource?

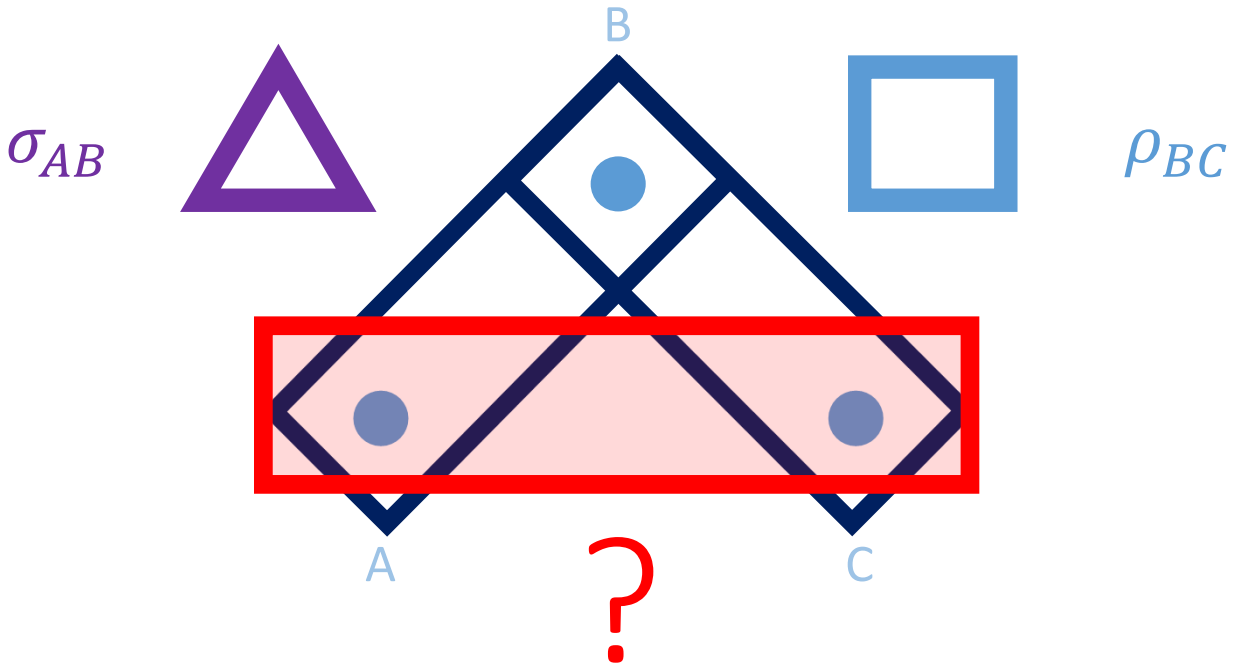


Entangled


σ_{AB}
B
ρ_{BC}
A
C

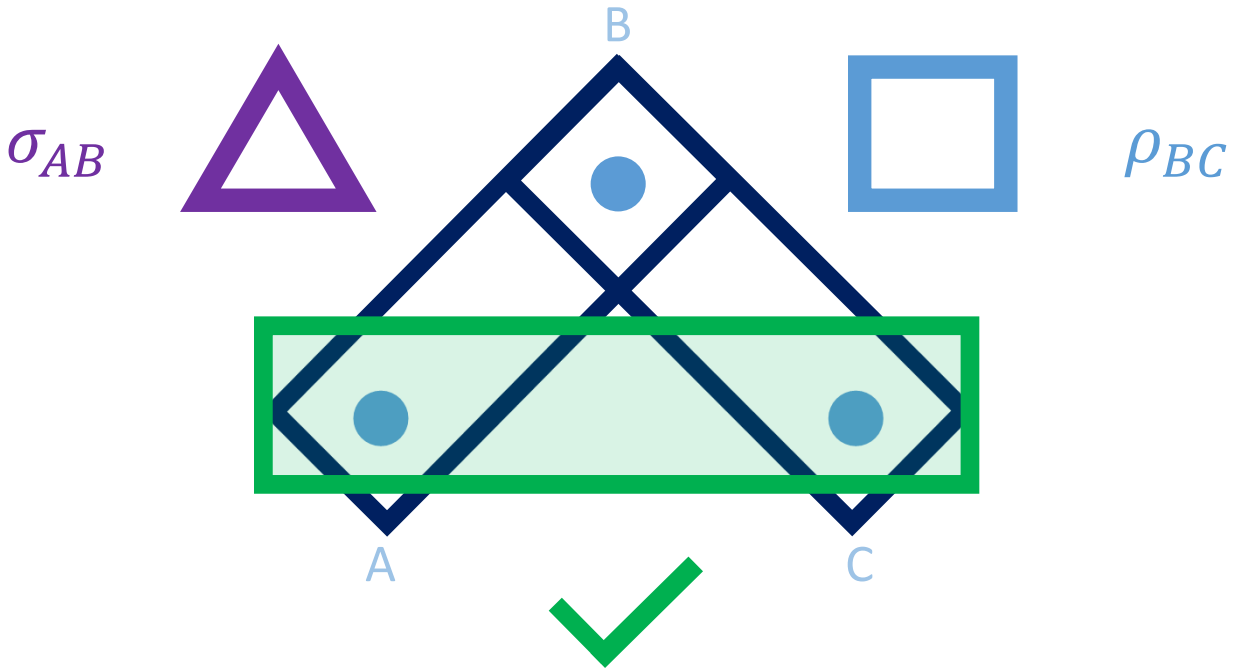


$$\forall\, \eta_{ABC} \text{ s.t. } \mathrm{tr}_A(\eta_{ABC}) = \rho_{BC}, \mathrm{tr}_C(\eta_{ABC}) = \sigma_{AB}$$



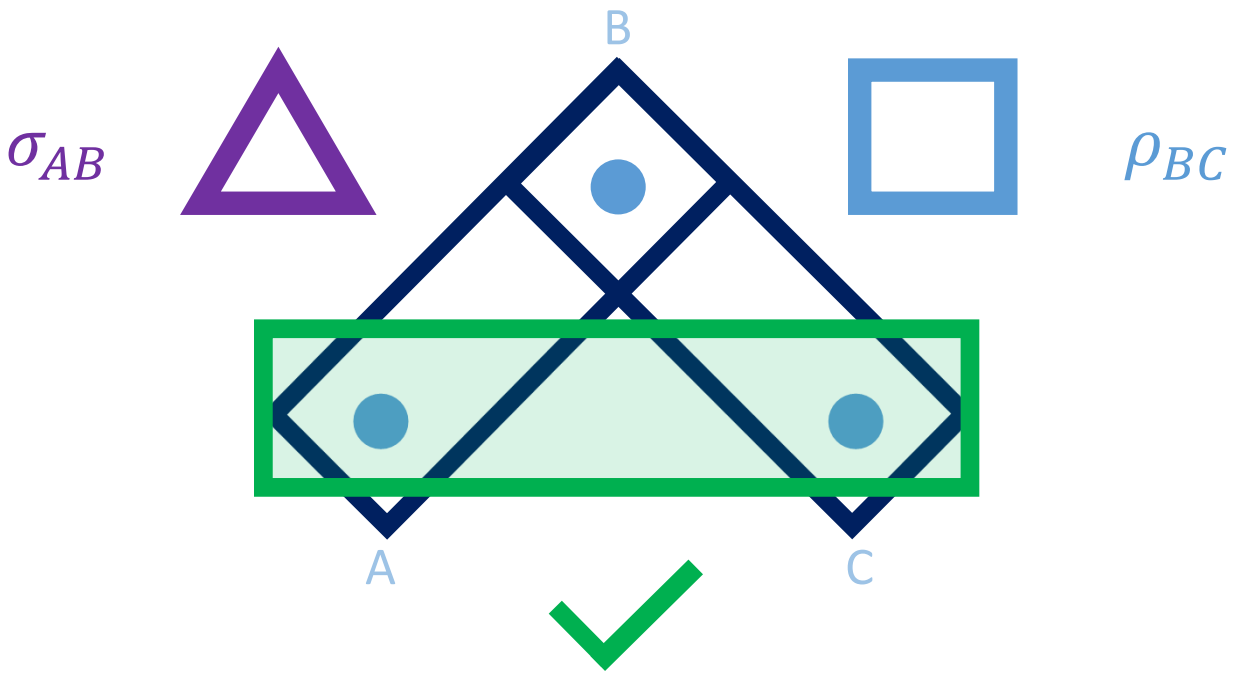
Entangled

$\forall\, \eta_{ABC}$ s.t. $\mathrm{tr}_A(\eta_{ABC}) = \rho_{BC}, \mathrm{tr}_C(\eta_{ABC}) = \sigma_{AB}$ ⟹ $\mathrm{tr}_B(\eta_{ABC})$

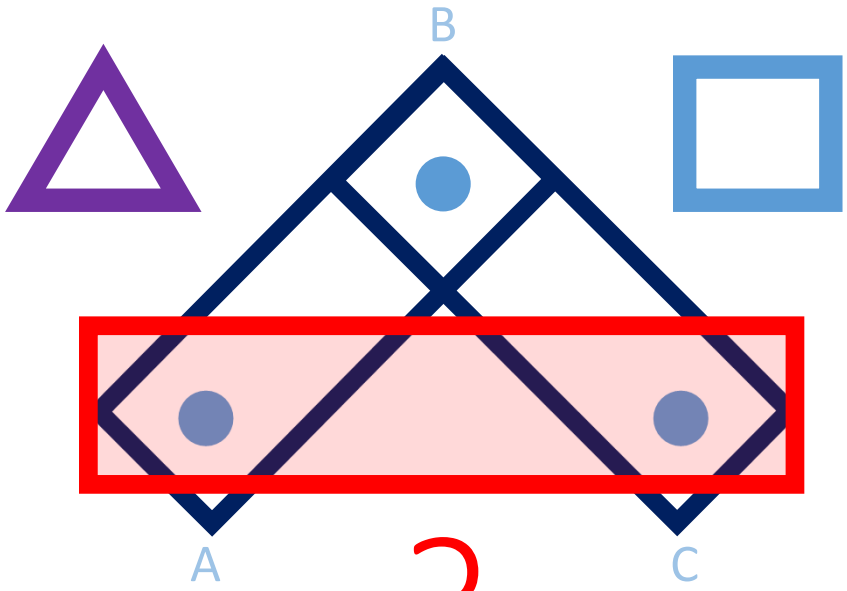


Entangled

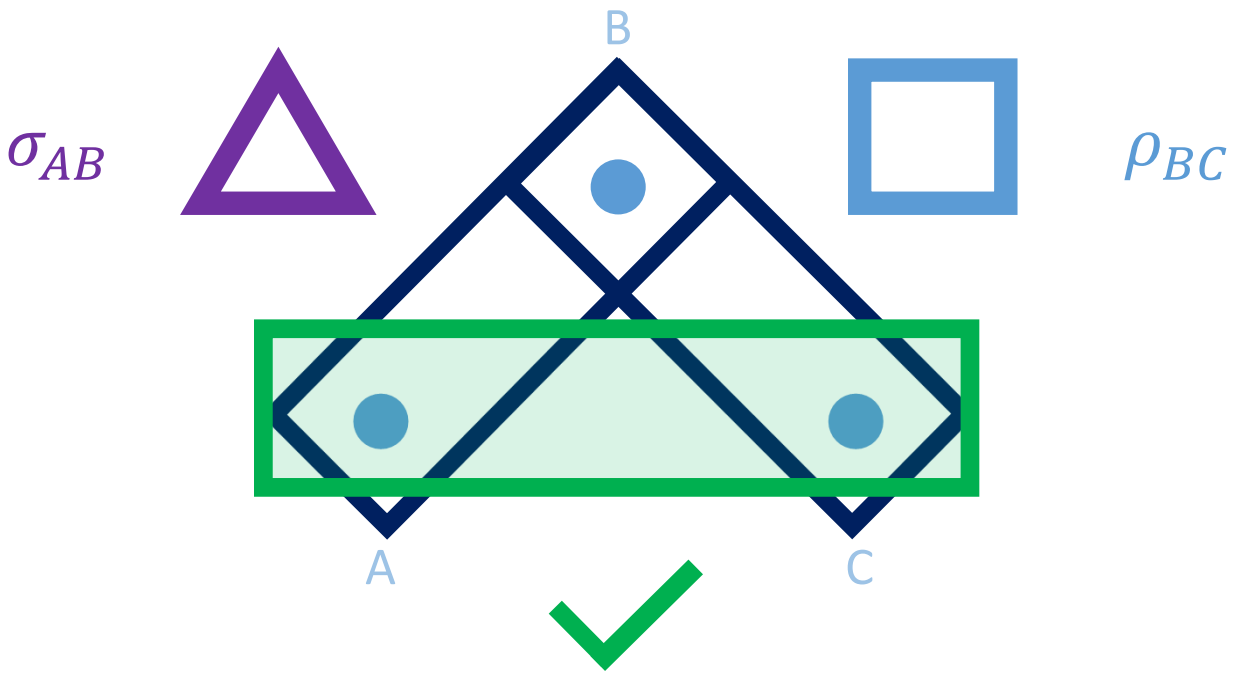
$\forall\, \eta_{ABC}$ s.t. $\mathrm{tr}_A(\eta_{ABC}) = \rho_{BC}, \mathrm{tr}_C(\eta_{ABC}) = \sigma_{AB}$ ⟹ $\mathrm{tr}_B(\eta_{ABC})$ is entangled



There exist compatible σ_{AB}, ρ_{BC} such that every η_{ABC} compatible with them must have entangled marginal in BC

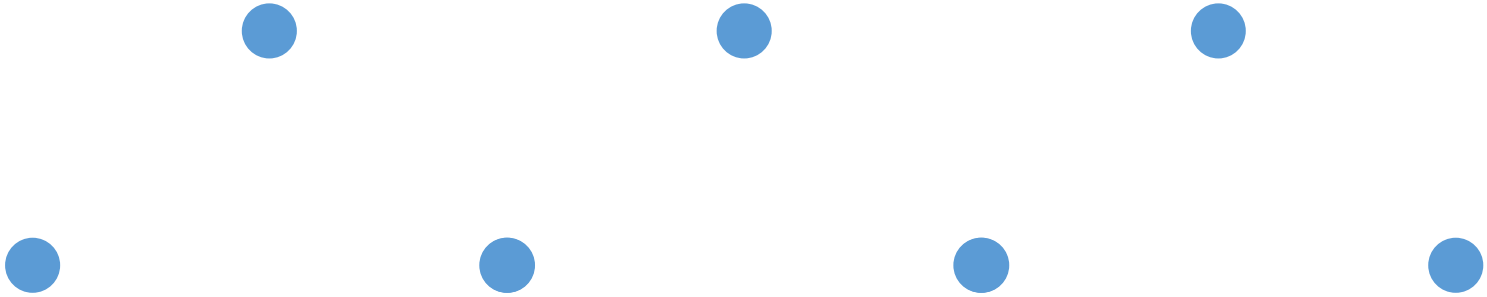


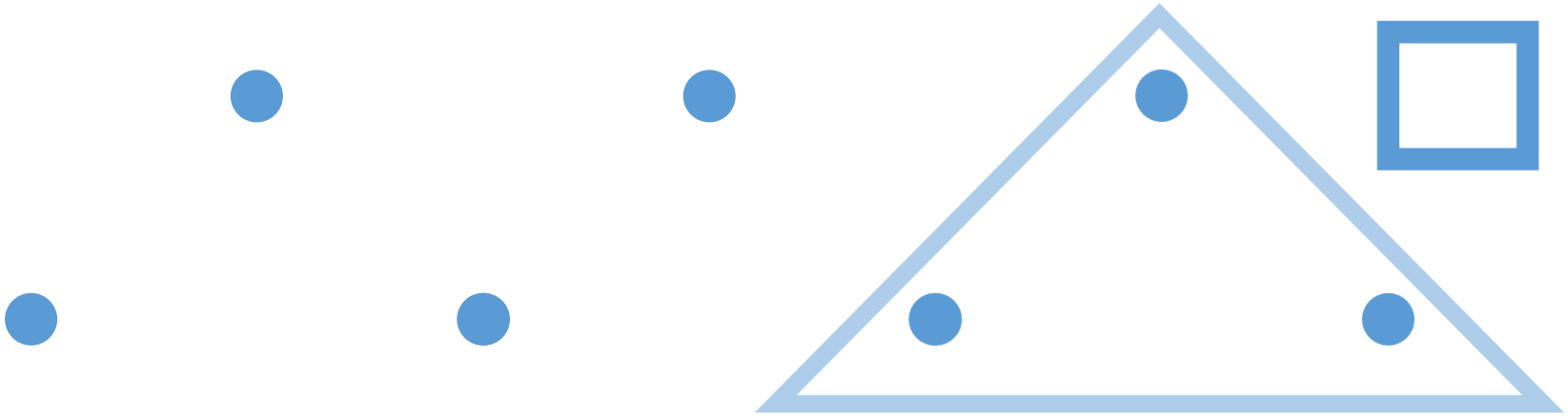
Nonlocal

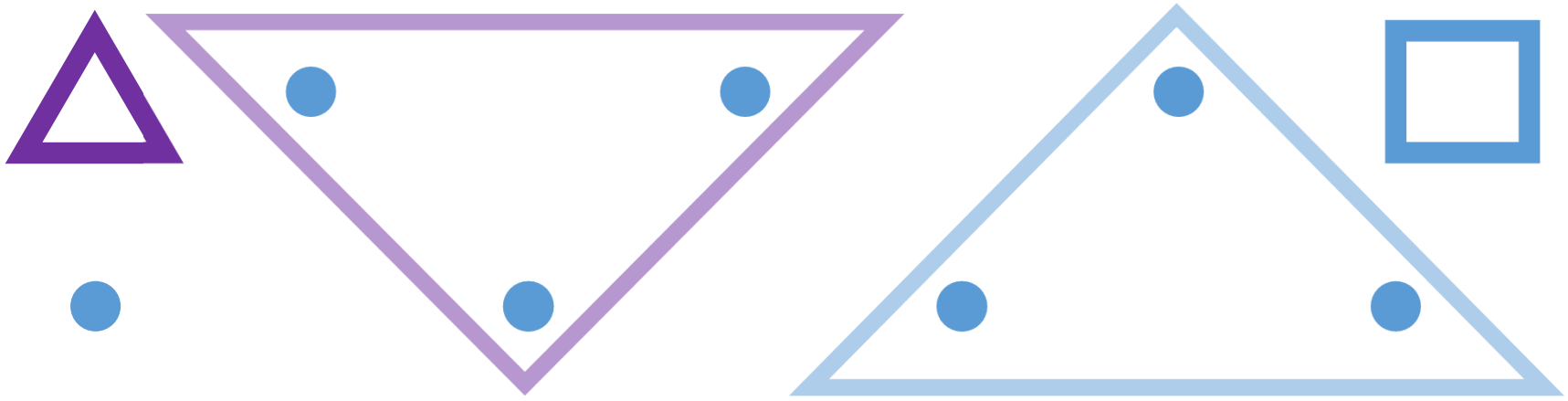


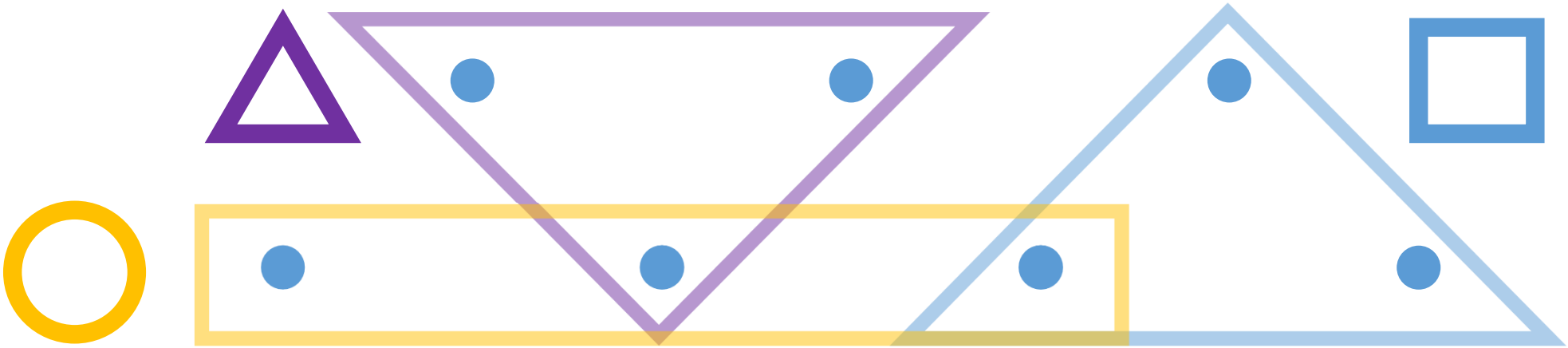
There exist compatible σ_{AB}, ρ_{BC} such that every η_{ABC} compatible with them must have nonlocal marginal in BC

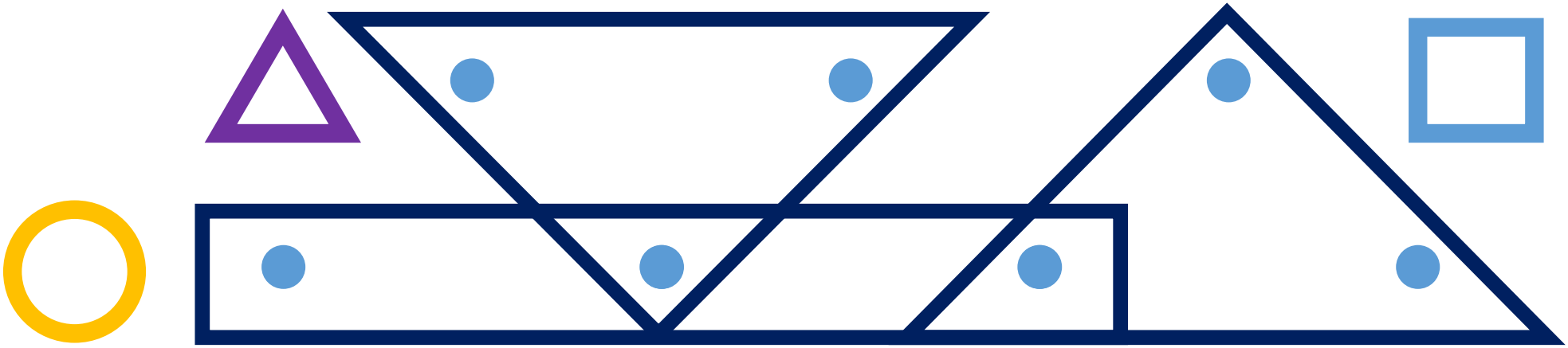

Marginal
Problems
Resource

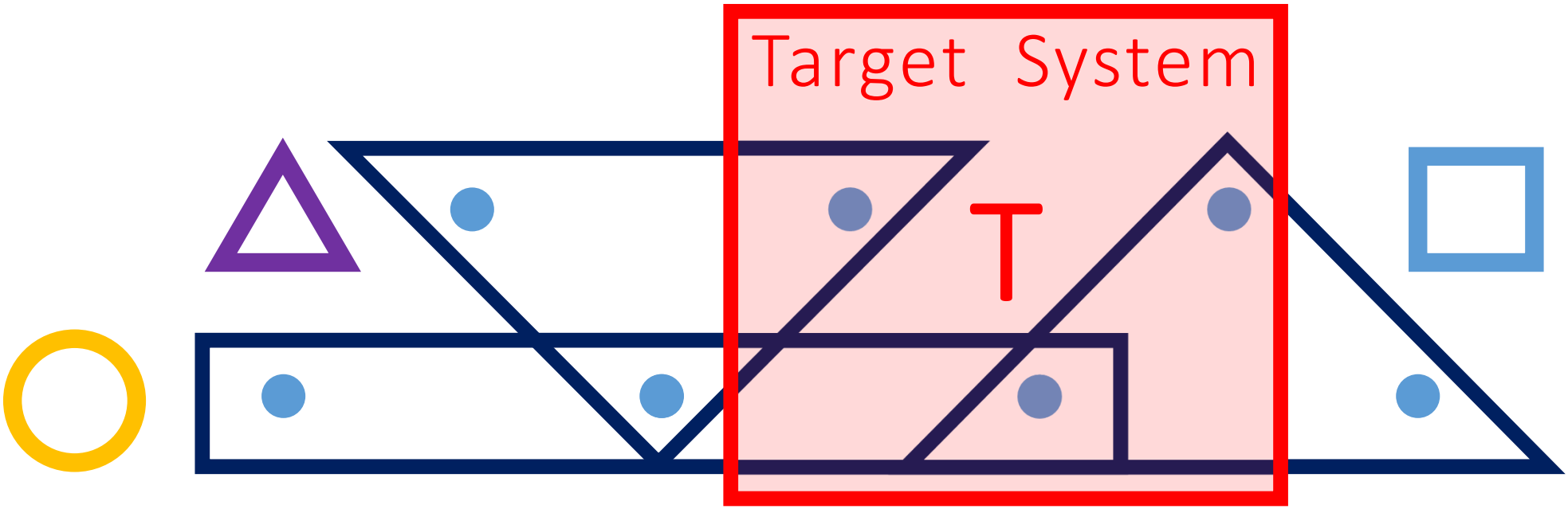


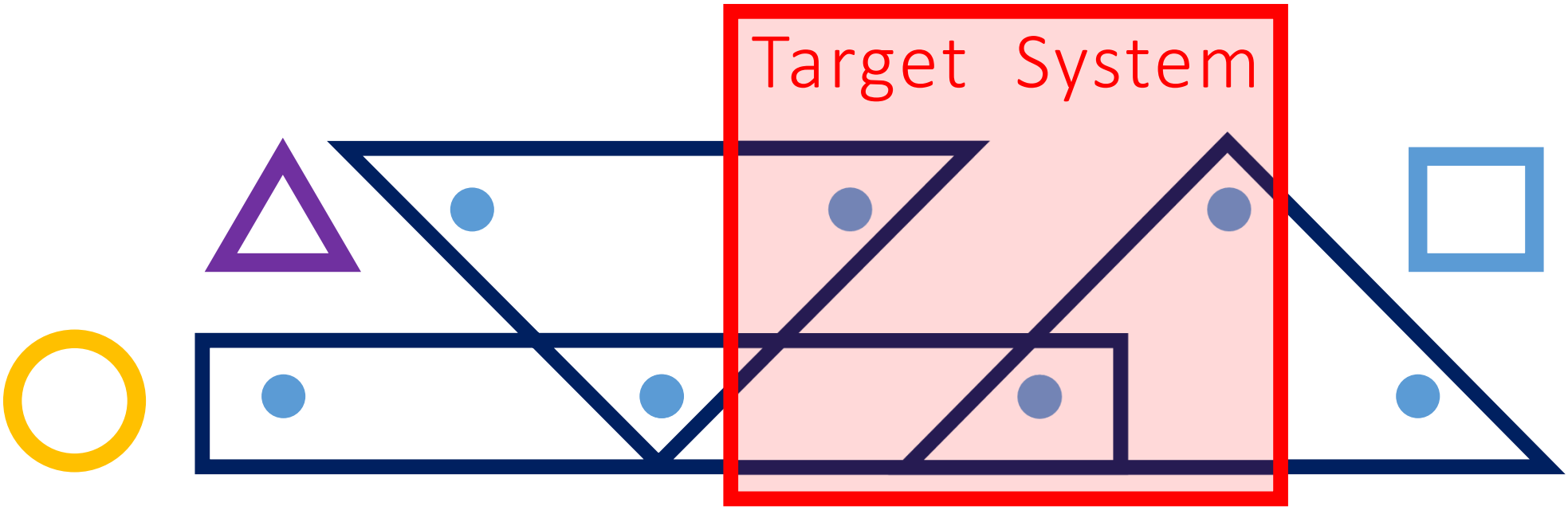




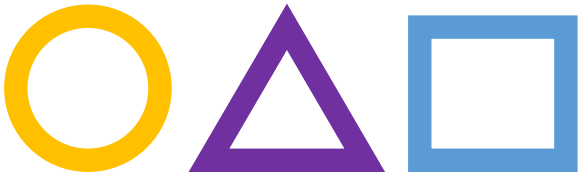





Target System
T

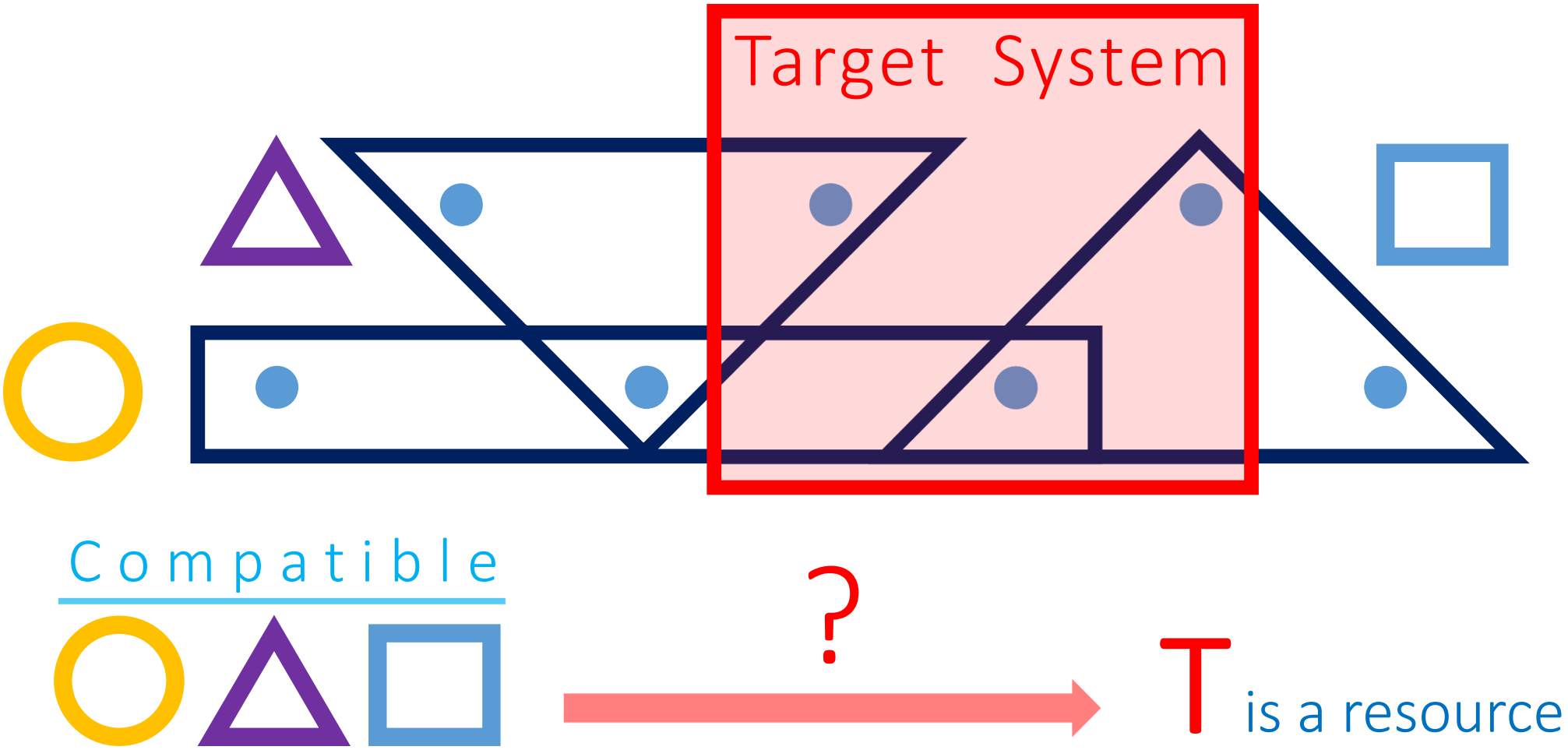


Compatible

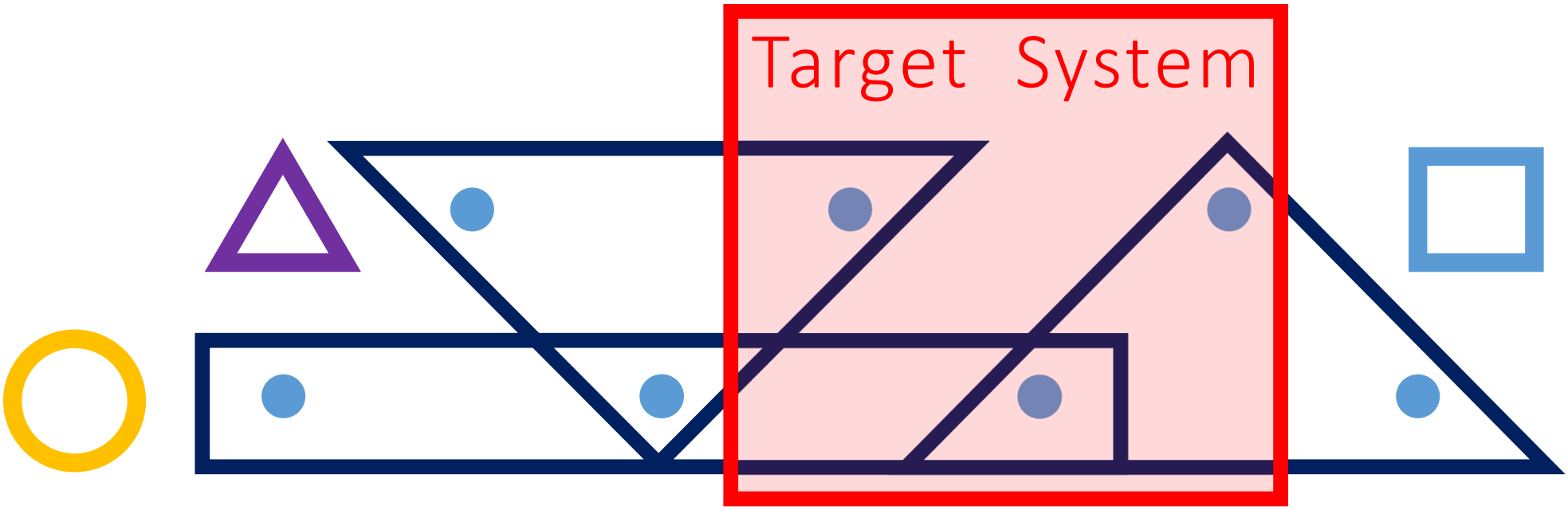


?

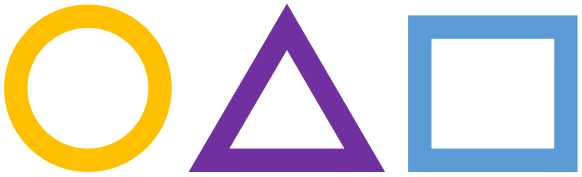
T is a resource



Resource Marginal Problems



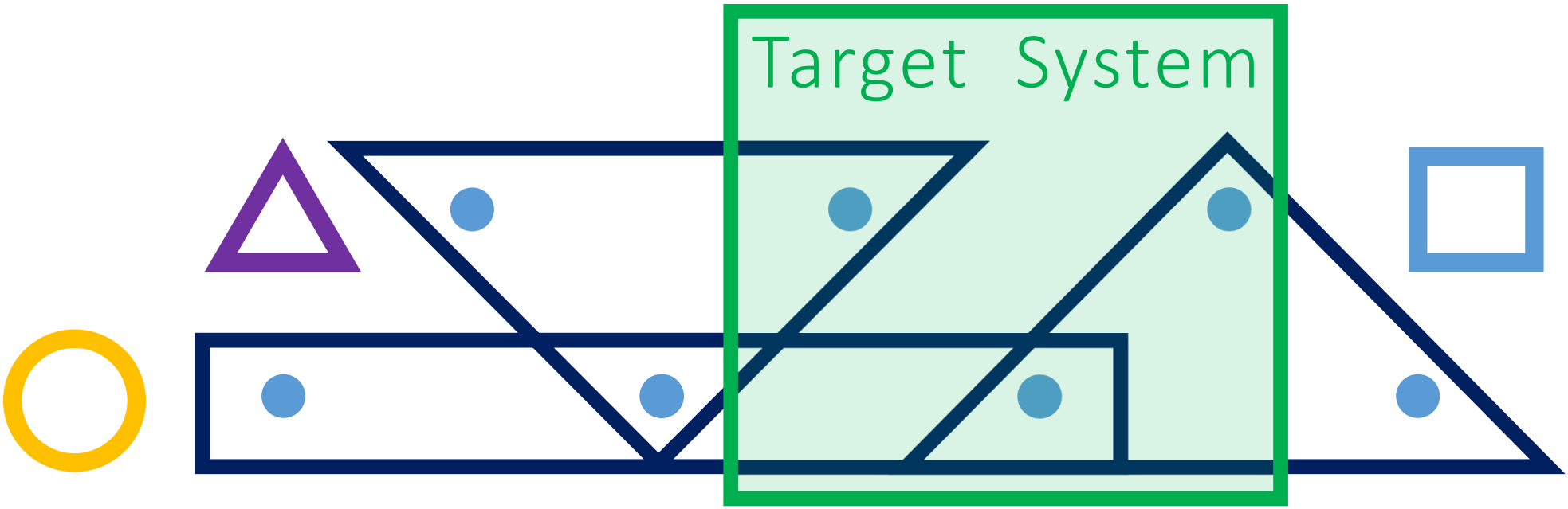
Compatible



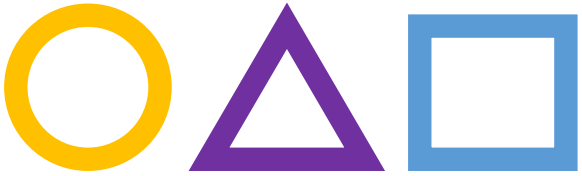
?

T is a resource

R()

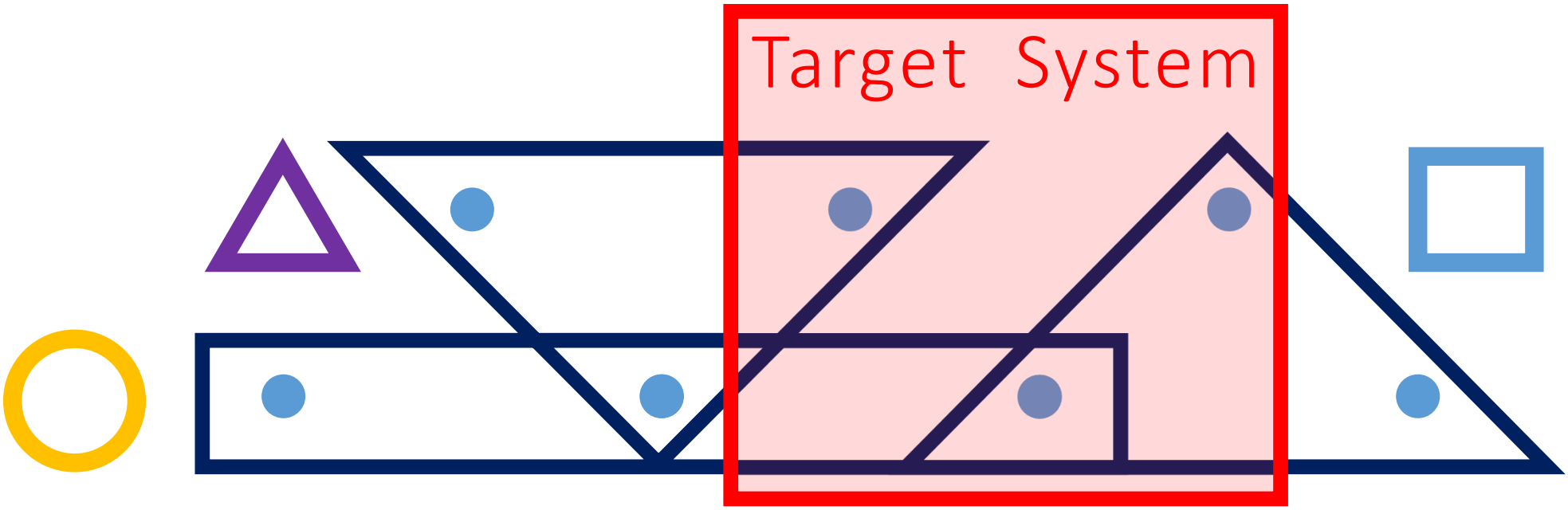


Compatible

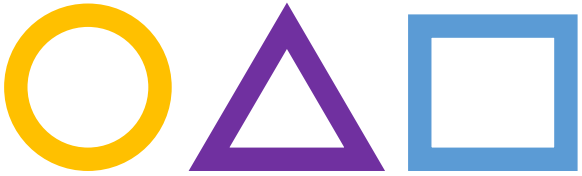


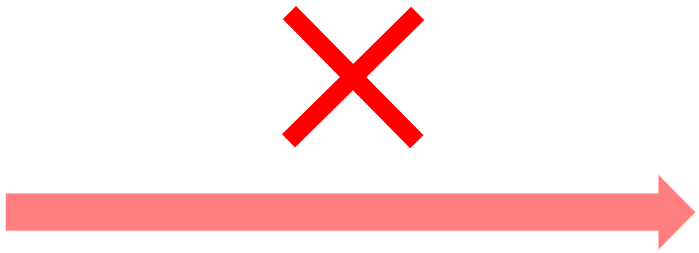
T is a resource

R( ) > 0



Compatible





T can be free

R( ) = 0

Take-Home Messages

Marginal Problems

Can they coexist?

Marginal Problems

Can they coexist?

 = Q Dynamics

Quantum Channel Marginal Problems

Quantum > Classical

Dynamical ≠ Static

Phys. Rev. Research **4**, 013249 (2022)

Marginal Problems

Can they coexist?

 = Q Dynamics

Quantum Channel Marginal Problems

Quantum > Classical

Dynamical ≠ Static

Phys. Rev. Research **4**, 013249 (2022)

Compatible

Can they certify target R?

Marginal Problems

Can they coexist?

 = Q Dynamics

Quantum Channel Marginal Problems

Quantum > Classical

Dynamical ≠ Static

Phys. Rev. Research **4**, 013249 (2022)

Compatible

Can they certify target R?

 = Q States & Q Dynamics

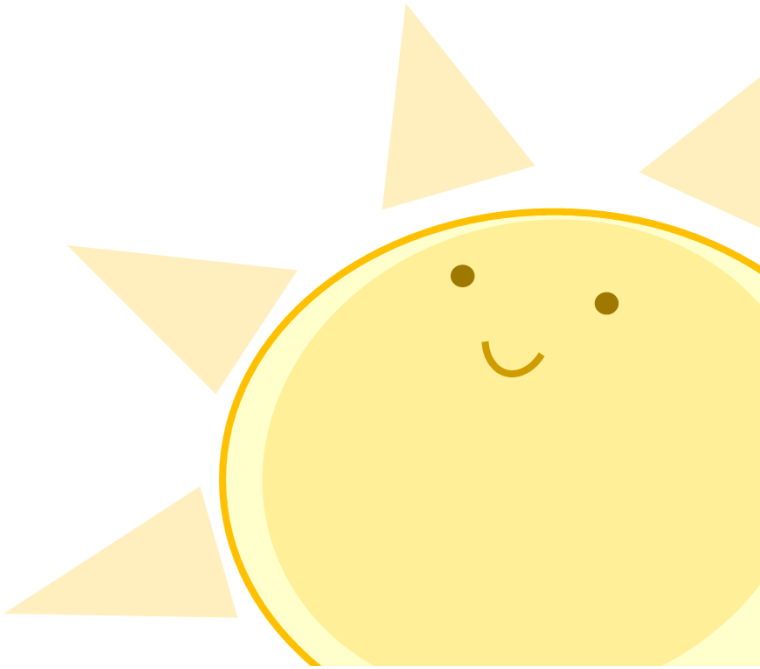
Resource Marginal Problems

R() ➡ Quantitative

Advantages in Discrimination Tasks

arXiv:2202.03523

Appendix



Appendix
What are Channels?

Channel

Channel

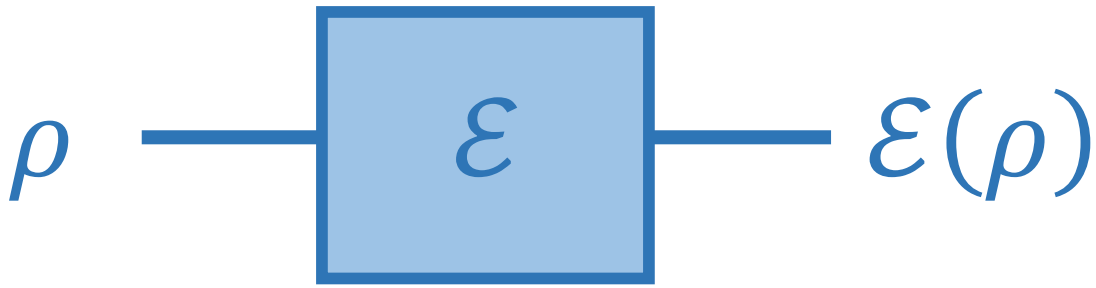
Completely-Positive Trace-Preserving Linear Map

Channel

Completely-Positive Trace-Preserving Linear Map

Channel

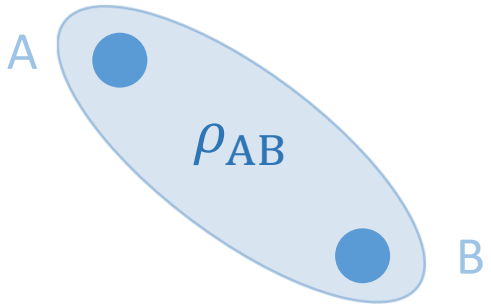
Completely-Positive Trace-Preserving Linear Map



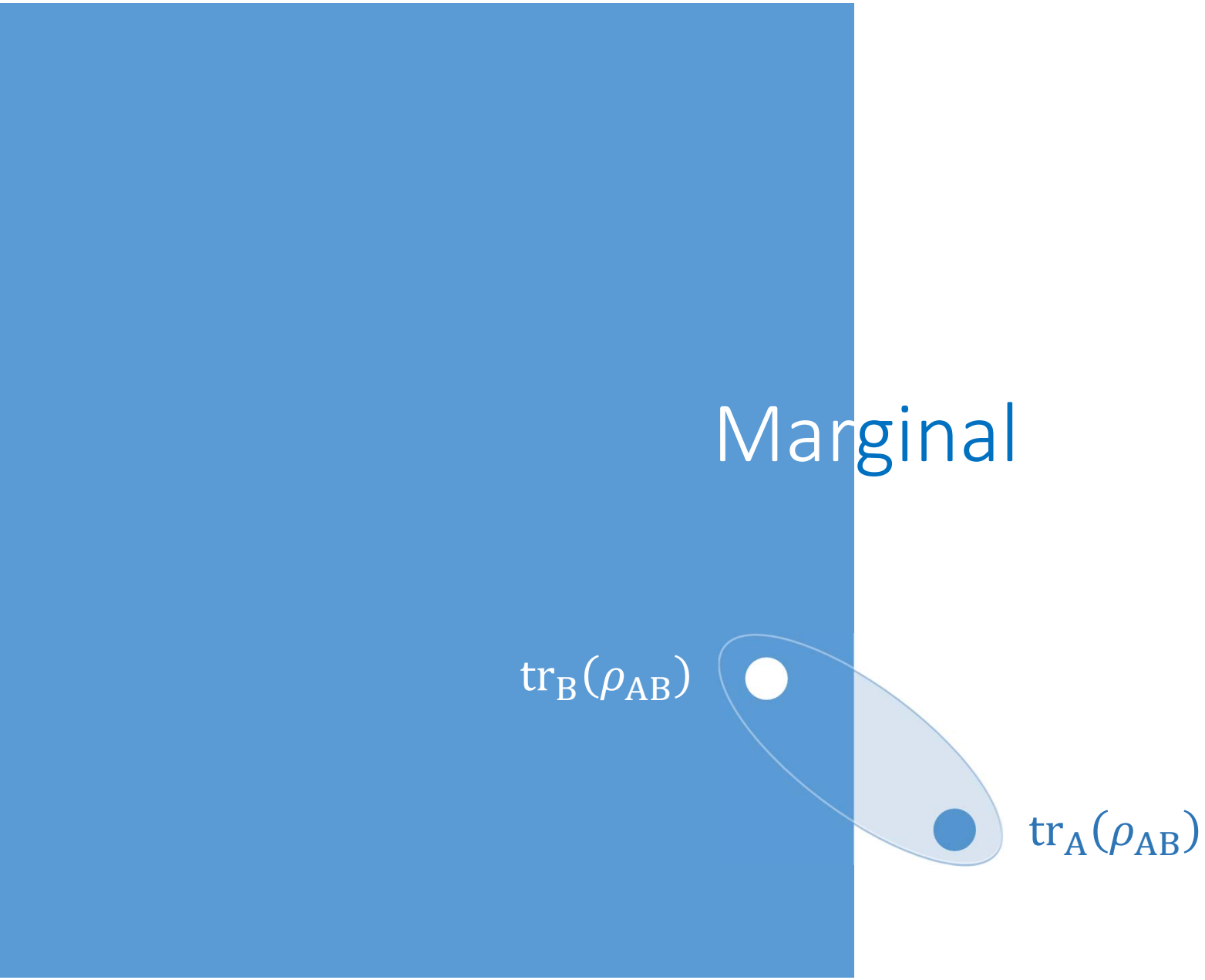
Appendix
What are Marginals?

Marginal

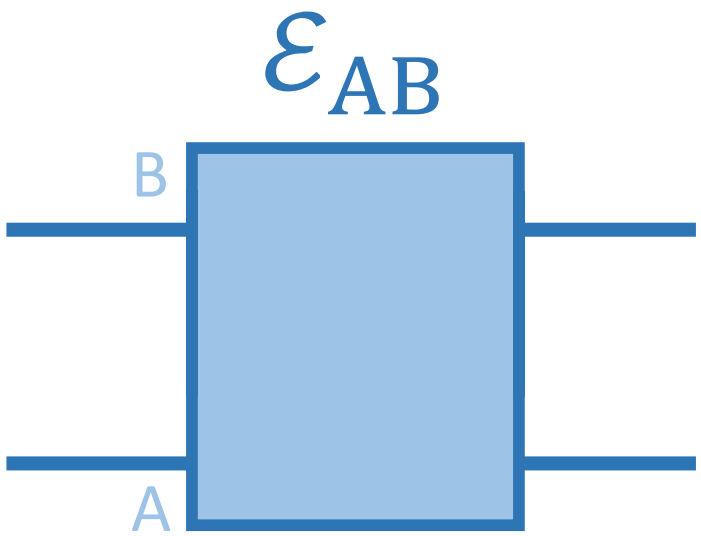
Marginal

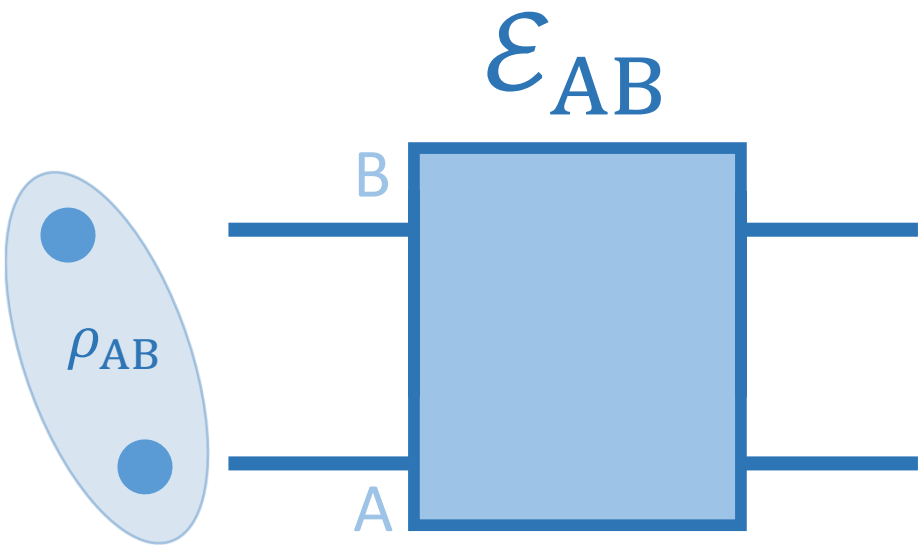


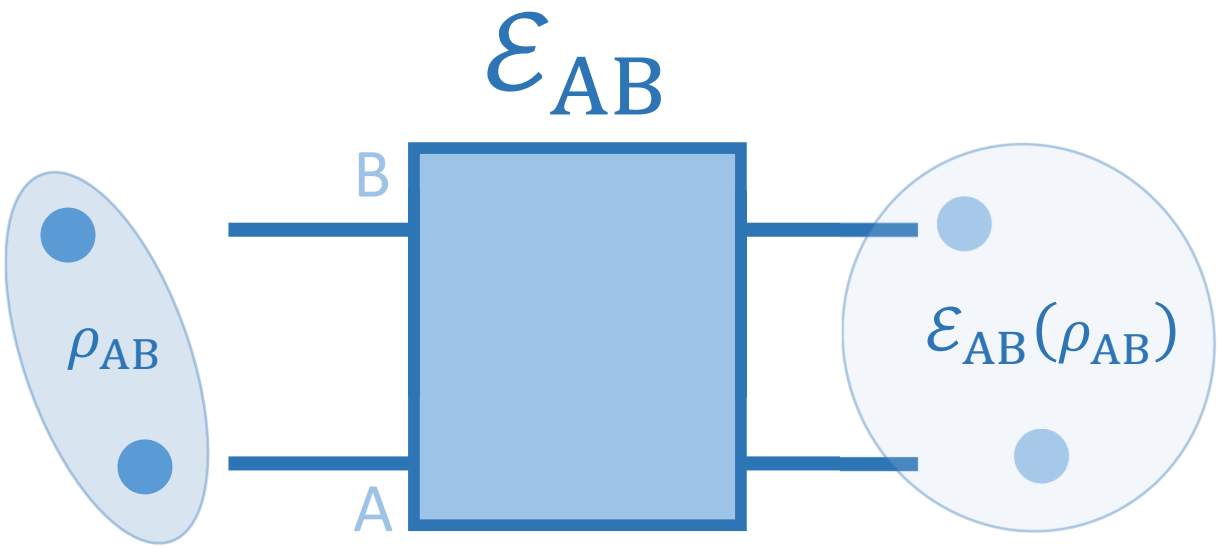
Marginal



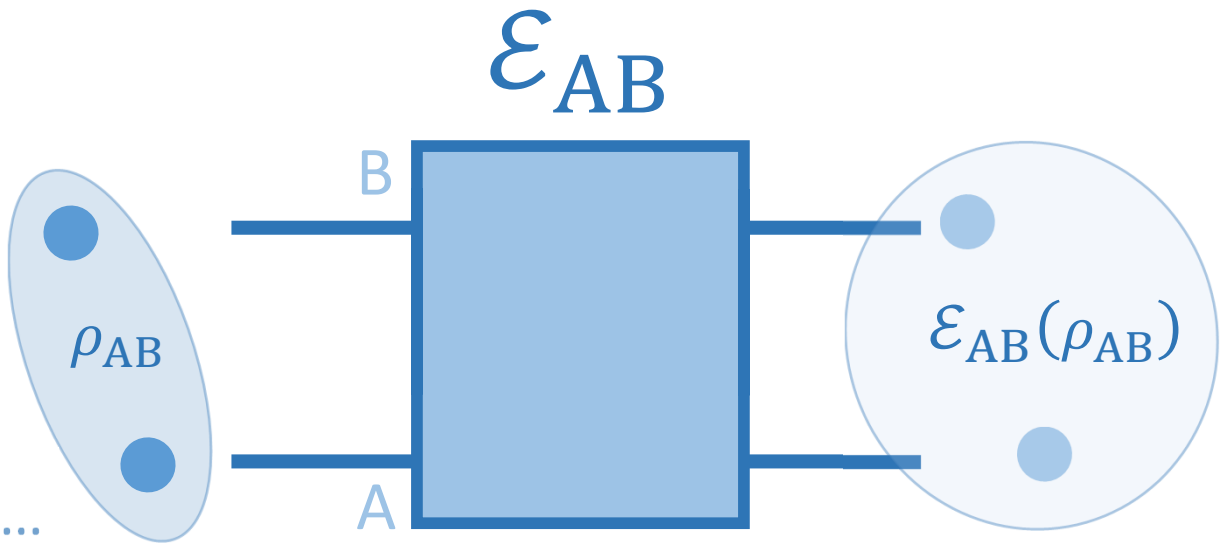
Marginal of Channel?

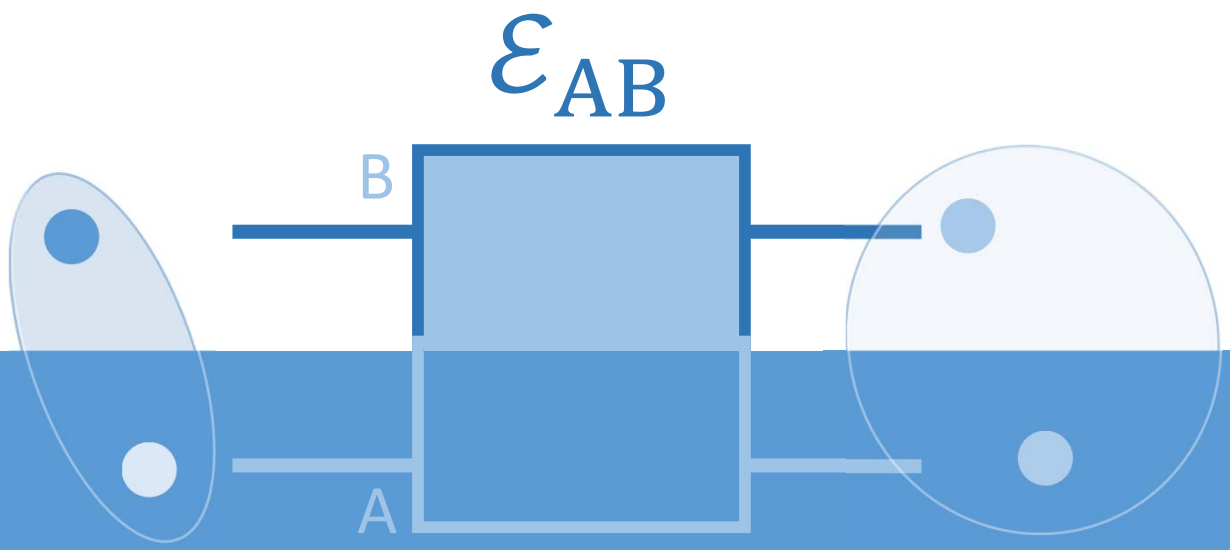

$\mathcal{E}_{AB}$
B
A


$\mathcal{E}_{\mathrm{AB}}$
B
A
ρ_{AB}


$\mathcal{E}_{AB}$
B
A
ρ_{AB}
$\mathcal{E}_{AB}(\rho_{AB})$

Local Agent will see…





$\mathrm{tr}_B(\rho_{AB})$

$\mathrm{tr}_B \circ \mathcal{E}_{AB}(\rho_{AB})$