Phenomenological thermodynamics of mutliple conserved quantities

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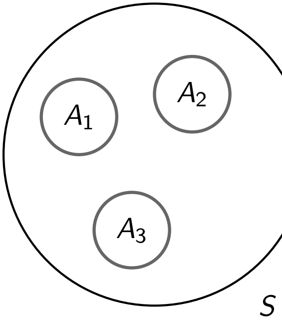
December 8, 2022

Why consider phenomenological thermodynamics

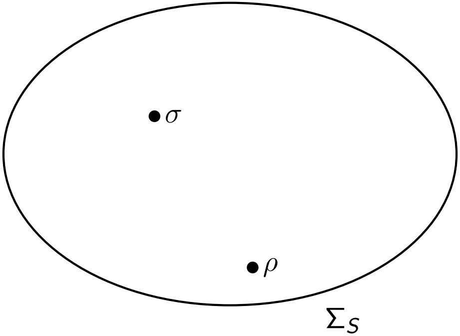
- Axiomatic framework makes assumptions clear[1].
- Multiple quantities make the role of information more explicit: Landauer's principle.
- No microscopic theory needed: Black holes.

[1] Philipp Kammerlander. "Tangible Phenomenological Thermodynamics". en. PhD thesis. ETH Zurich, 2019. DOI: 10.3929/ETHZ-B-000413414, Philipp Kammerlander and Renato Renner. "Tangible phenomenological thermodynamics". In: (Feb. 20, 2020). arXiv: 2002.08968 [math-ph]. URL: https://arxiv.org/abs/2002.08968.

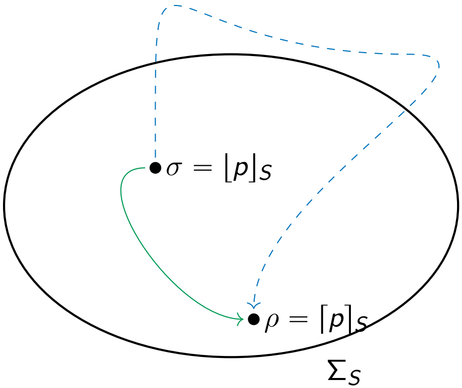
A **system** is a non-empty and finite subset of the thermodynamic world $\mathcal{S}$



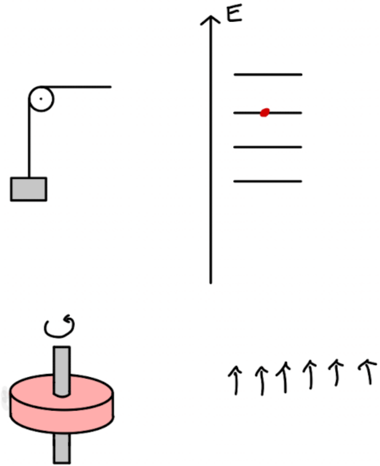
The **state space** of the system is Σ_S



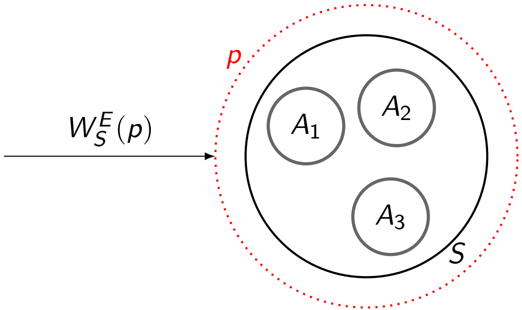
Process $p \in \mathcal{P}$



Work = Quantity + Information

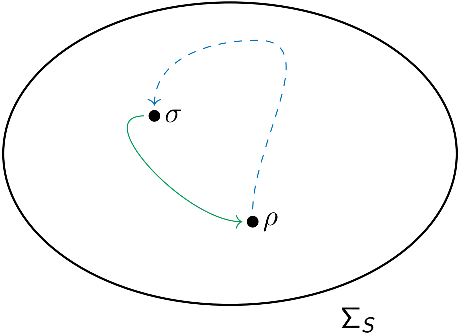


$W_S^B(p)$: work of quantity B done by process p on system S.



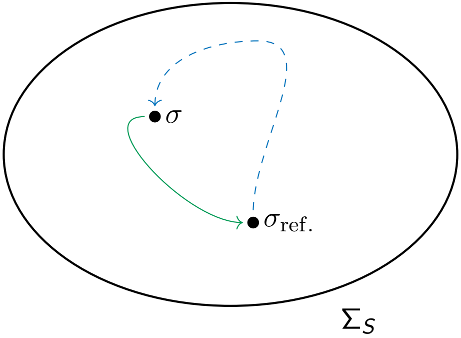
First law

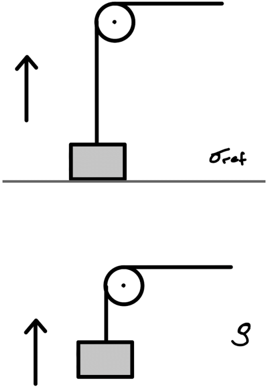
- There is a non-dissipative process connecting any two states in Σ_S.



Internal B-Worth

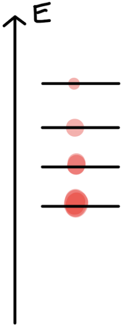
- For p a work process on a system S $U_S^B(\sigma) = +/-W_S^B(p) + U_S^B(\sigma_{\text{ref.}})$





Heat = quantity

$$Q_S^B(p) = \Delta U_S^B - W_S^B(p)$$



Bath: Simple, Boundless, Translation invariant, Passive

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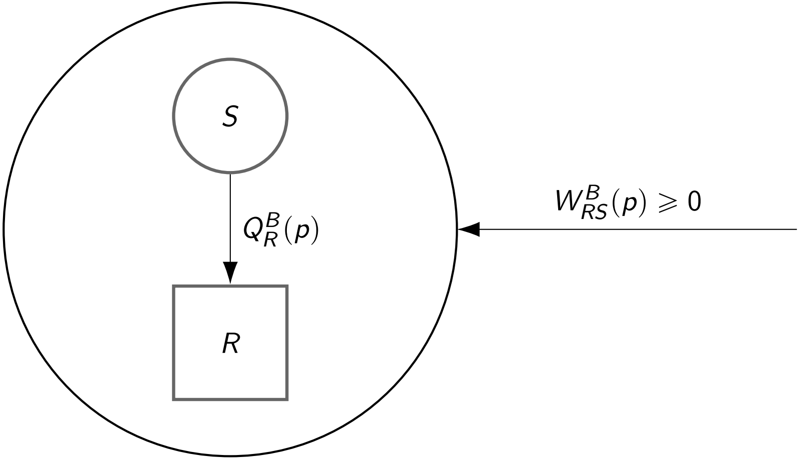
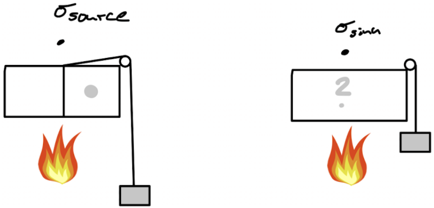
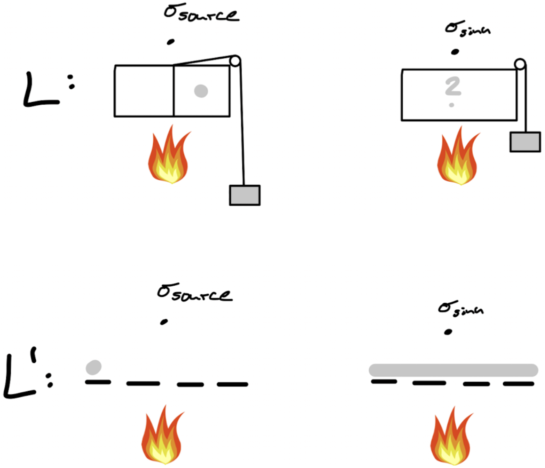


What about just information?

L:



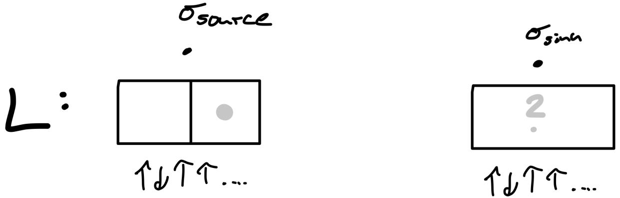
$$W_L^E(p_\ell) = -k_E T \log(2)$$



$$W_L^E(p_\ell) = -k_E T \log(2)$$

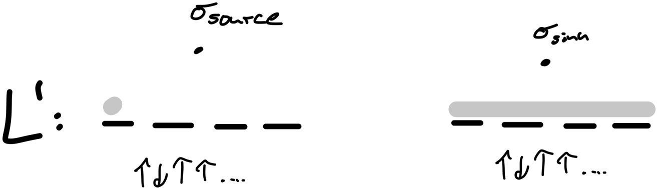
$$\frac{S(L)}{S(L')} = \frac{W_L^E(p_\ell)}{W_{L'}^E(p_{\ell'})}$$

$$W_{L'}^E(p_{\ell'}) = -k_E T \log(4)$$



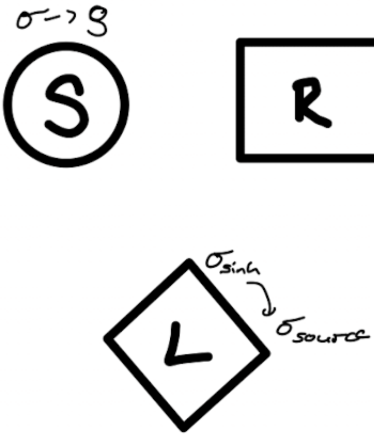
$$W_L^{\text{ang.}}(p_\ell) = -k_{\text{ang.}} T log(2)$$

$$\frac{S(L)}{S(L')} = \frac{W_L^{\text{ang.}}(p_\ell)}{W_{L'}^{\text{ang.}}(p_\ell')}$$

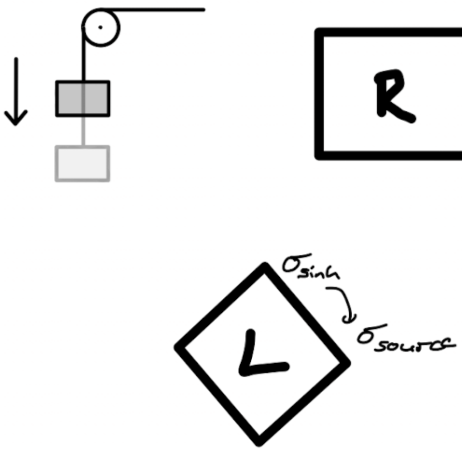


$$W_{L'}^{\text{ang.}}(p_\ell') = -k_{\text{ang.}} T log(4)$$

Entropy from Landauer boxes



Entropy from Landauer boxes



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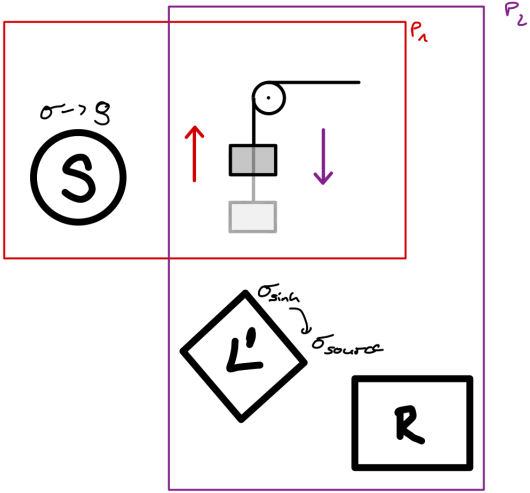
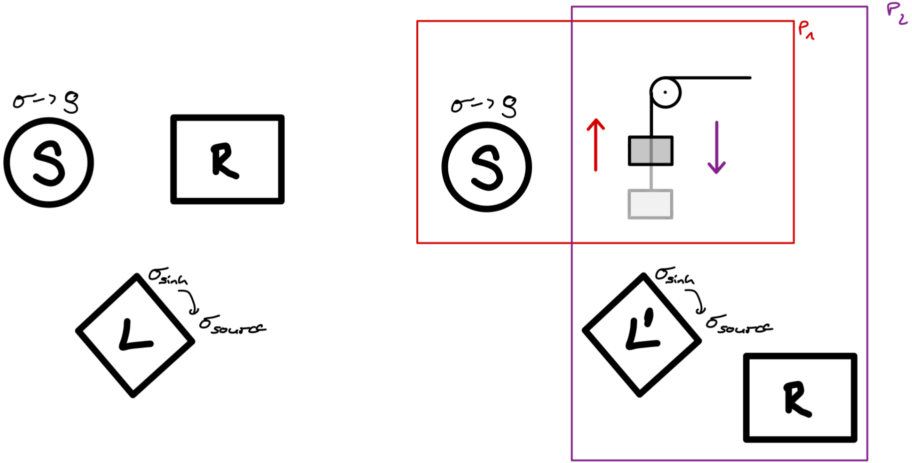


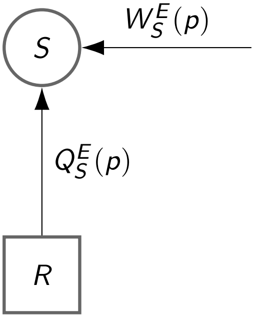
Entropy from Landauer boxes



$$\Delta S(\sigma \to \rho) = S(L) - S(L')$$

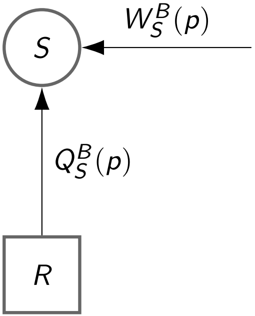
Recovers the usual definition of entropy

$$\Delta S(\rho \to \sigma) = \frac{Q_S^E(p)}{T_R} \quad \backsim \quad dS = \frac{dU - dW}{T}$$



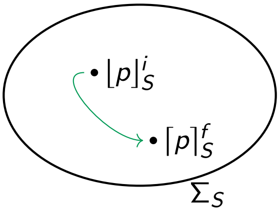
Recovers the usual definition of entropy

$$\Delta S(\rho \to \sigma) = \sum_B \frac{Q_S^B(p)}{k_B T_R^B}$$



Entropy always increases

$$S(\lfloor p \rfloor_S^i \to \lceil p \rceil_S^f) \geqslant 0$$



Recover results of thermodynamics

- Generalized second law $\sum_B \frac{W_S^B(\boldsymbol{p})}{k_B T_R^B} \geqslant 0$
- Generalized Clausius theorem
- Generalized Carnot theorem
- Generalized zeroth law

Application: Black holes

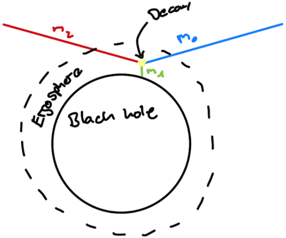
Systems and processes: black hole

- State space: $\Sigma_{BH} = \{(M, L)|\ M^2 \geqslant L^2/M^2\}$

- Dump mass m from infinity

- Initial state: (M_1, L_1)
- Final state: $(M_1 + m, L_1)$

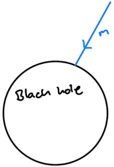
- Reversible Penrose process:



- Initial state: (M_1, L_1)
- Final state: $(M_1 + m_1, L_1 + \ell_1)$
- Irreducible mass is constant
$M_{1/2}^2 = M_{irr}^2 + \frac{L_{1/2}^2}{4M_{irr}^2}$

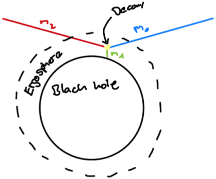
First law: black hole

- Dump mass m from infinity:



- Initial state: (M_1, L_1)
- Final state: $(M_1 + m, L_1)$
- $W_{E,BH} = m,\ W_{L,BH} = 0$

- Reversible Penrose process:

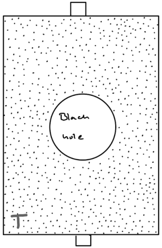


- Initial state: (M_1, L_1)
- Final state: $(M_1 + m_1, L_1 + \ell_1)$
- Irreducible mass is constant
  $M_{1/2}^2 = M_{irr}^2 + \frac{L_{1/2}^2}{4M_{irr}^2}$
- $W_{E,BH} = m_1,\ W_{L,BH} = \ell_1$

$$U_{E,BH} = M + const.,\ U_{L,BH} = L + const.$$

Entropy: black hole

- We can use the reversible Penrose process
- Need a process that changes the irreducible mass M_{irr} of the black hole.
- Use isothermic process considered by Kaburaki and Okamoto[2]



$$\Delta S = 4\pi \left(M_{2,irr}^2 - M_{1,irr}^2\right)$$
$$= (A_2 - A_1)/4$$

[2] Osamu Kaburaki and Isao Okamoto. "Kerr black holes as a Carnot engine". In: *Physical Review D* 43.2 (1991), pp. 340–345. DOI: 10.1103/physrevd.43.340.

Conclusion

- Extended the phenomenological thermodynamic framework to multiple quantities.
- Defined thermodynamic entropy by converting between quantities using Landauer's principle.
- A black hole fits into the the thermodynamic framework.
- We find the expected internal energy, angular momentum and entropy using the thermodynamic framework.