Quantum Coherence and Unitary Work Extraction

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based on **PRL 125, 180603 (2020)** with
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Quantum Resources Workshop

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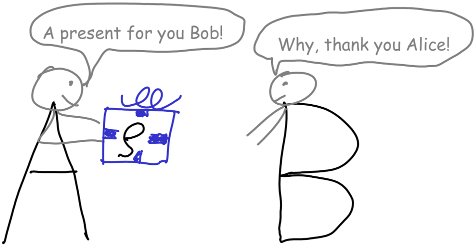
- A broad question:
  What is *quantum* in Quantum Thermodynamics?
- A more specific question:
  How much work can be extracted from a quantum state ρ (with respect to a Hamiltonian H)?



Alice presents Bob with a quantum state.
How much work can Bob extract?

Thomson's 2nd law

"No work can be extracted from a closed equilibrium system during a cyclic variation of a parameter by an external source"

[Allahverdyan & Nieuwenhuizen, Physica A **305**, 542 (2002)]

Rules of the game

Cyclicity:

$$\tilde{H}(t) = H + V(t)$$

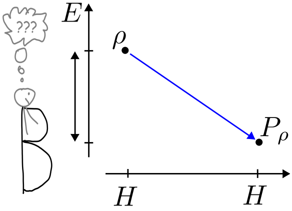
with $V(t) = 0$ for $t < 0$ or $t > \tau$

Unitarity:

$$\langle W \rangle = \mathrm{tr}[\rho H_0] - \mathrm{tr}[\rho' H_0]$$

Ergotropy

is the maximum work extractable under cyclic, unitary evolution.



$$\rho = \sum r_n |r_n\rangle\langle r_n|\,,\ \text{with } r_n \geq r_{n+1} \forall n$$

$$H = \sum \epsilon_n |\epsilon_n\rangle\langle \epsilon_n|\,,\ \text{with } \epsilon_n \leq \epsilon_{n+1} \forall n$$

Ergotropy & passive states

$$\mathcal{E}(\rho) = \mathrm{tr}\left[H\left(\rho - P_\rho\right)\right]$$

$$P_\rho = \min_U U\rho U^\dagger = \sum r_n |\epsilon_n\rangle\langle\epsilon_n| \qquad (\text{e.g. } r_n \propto \exp\left[-\beta\epsilon_n\right])$$

$$\lim_{n\to\infty} \rho^{\otimes n} : \mathcal{E} \to F_{n.e.} := D(\rho || \rho_{\beta^*})$$

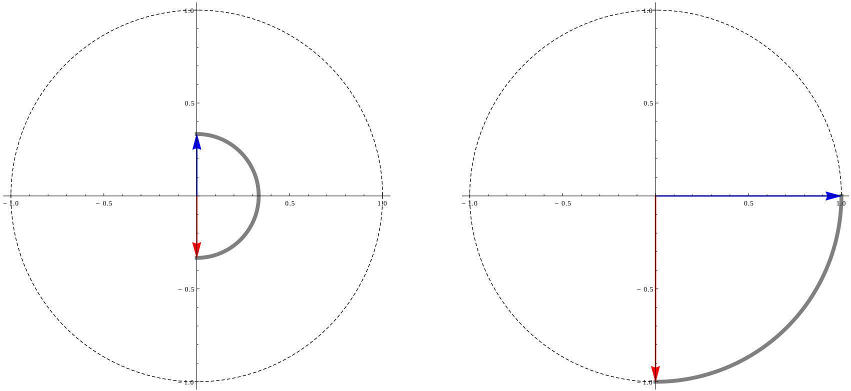
[Pusz & Woronowicz, Comm. Math. Phys. **58**, 273 (1978); Lenard, J. Stat. Phys. **19**, 575 (1978)]

[Allahverdyan, Balian & Nieuwenhuizen, EPL **67**, 565 (2004)]

Simplistic illustration for a qubit

Ergotropy quantifies work extraction from population inversion and coherence.

$$H = -\sigma_z$$



Coherence: resource-theoretic description

Quantum resource theory of coherence (in a nutshell)

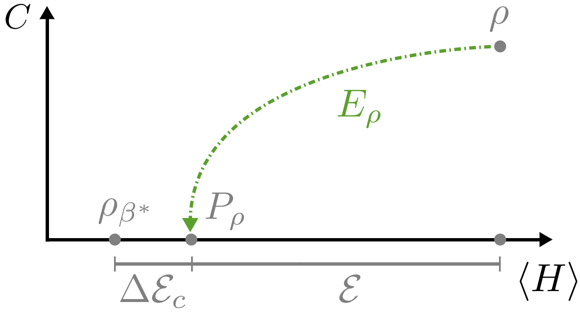
- fixed basis $\{|j\rangle\}$
  here: $|j\rangle = |\epsilon_j\rangle$
- free states: $\phi = \sum_j p_j |j\rangle\langle j|$
  no off-diagonals in given basis
- free operations (SIO*): $K_n = \sum_m e^{i\varphi_m} |\pi_m\rangle\langle m|$
  unitaries: $U = \sum_m e^{i\varphi_m} |\pi_m\rangle\langle m|$ *with* π *invertible*
  $\Rightarrow$ *permutations and phases* $\Rightarrow$ *no creation of coherence*
- monotone: $C(\rho) = \min_\phi D(\rho||\phi) = S(\Delta[\rho]) - S(\rho)$
  $C(\rho)$ *measures coherence and cannot increase under SIOs*

*SIO: Strictly Incoherent Operations

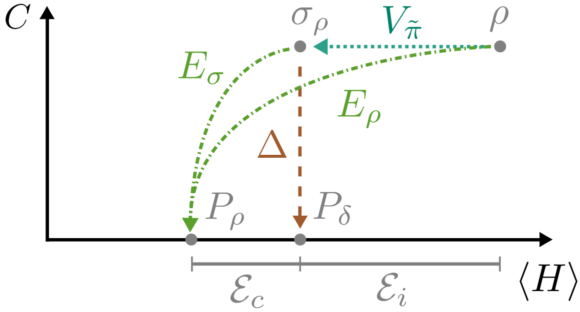
[Winter & Yang, PRL **116**, 120404 (2016); Yadin et al., PRX **6**, 041028 (2016)]

[Streltsov, Adesso & Plenio, RMP **89**, 041003 (2017)]

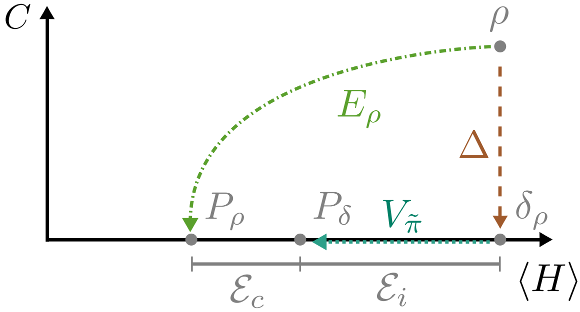
Coherent ergotropy



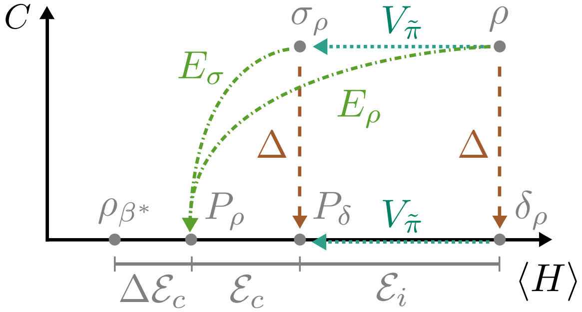
Coherent ergotropy



Coherent ergotropy



Coherent ergotropy



$$\beta\mathcal{E}_c = C(\rho) + D(P_\delta || \rho_\beta) - D(P_\rho || \rho_\beta)$$

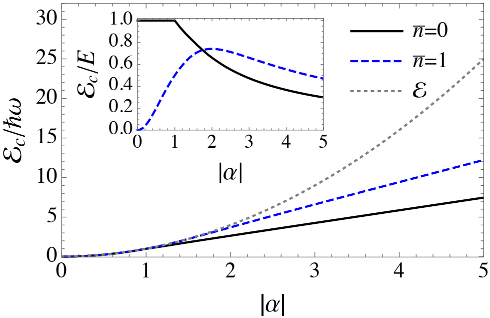
Coherent ergotropy: bounds

$$C(\rho) - D(P_\rho || \rho_\beta) \leq \beta \mathcal{E}_c(\rho) \leq C(\rho) + D(P_\delta || \rho_\beta)$$

Upper bound

Saturated for $\beta^* = \beta$ if $\rho = U \rho_{\beta^*} U^\dagger$, e.g.:

- qubits
- Gaussian states



$\rho = D(\alpha) \rho_\beta D^\dagger(\alpha)$, with

$$D(\alpha) = e^{\alpha a - \alpha^* a^\dagger}$$

Summary and ergotropy in context

Summary

- ergotropy: coherent and incoherent part
- coherent ergotropy: entropic expression

Perspective

- work in quantum thermo: task-dependent

[Niedenzu et al., Quantum **3**, 195 (2019)]

- coherence and ergotropy in NESS
- quantum engines: characterisation of load

[von Lindenfels et al., PRL **123**, 080602 (2019), Horne et al., npj:QI **6**, 37 (2020)]

PRL 125, 180603 (2020) [arXiv:2006.05424]

with G. Francica, M. Mitchison, G. Guarnieri, J. Goold, F. Plastina

Thank you for your attention!

- Work extraction from unknown sources: **arXiv:2209.11076**
- PhD position available
- MSc in Quantum Science & Technology: **tcd.ie/physics/quantumtech/**

Quantum Info @Trinity College Dublin

Alessandro Candeloro (joining in '23) | Alec Boyd | Felix Binder | Marco Radaelli | Magdalini Zonnios | Simon Milz (joining in '23)

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