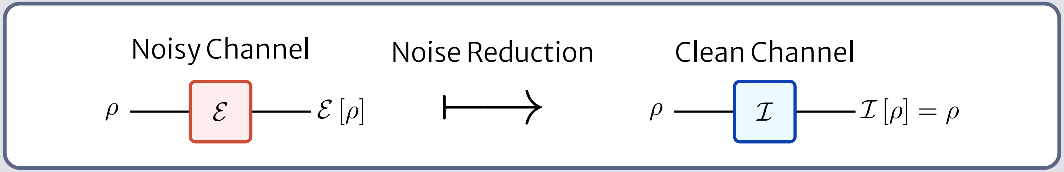
Resource Theories and Noise Reduction

Graeme Berk, Simon Milz, Felix Pollock, and Kavan Modi

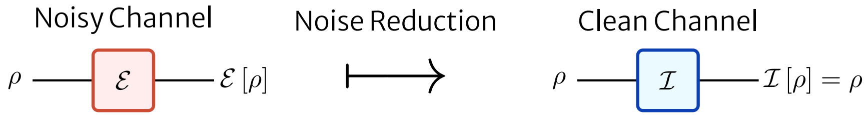
5th of December 2022

Synopsis

Noise Reduction as a Resource Transformation

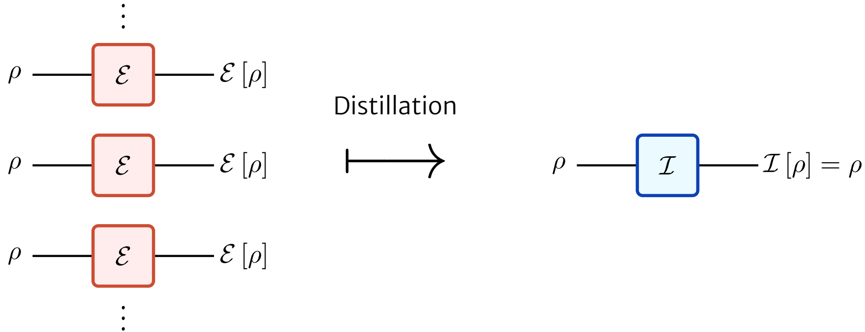


Noise Reduction as a Resource Transformation



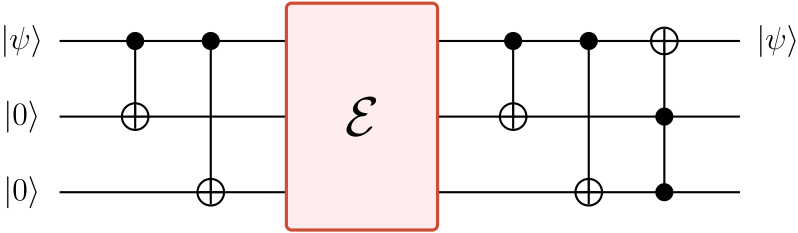
Golden rule for quantum resource theories: free transformations cannot increase resource value.

Noise Reduction as Resource Distillation



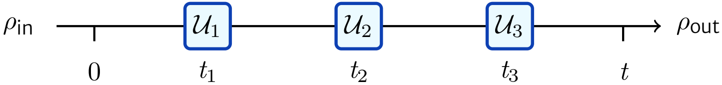
Noise Reduction as Resource Distillation

Bit flip code



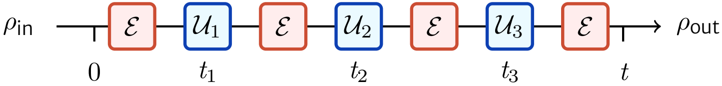
Dynamical Decoupling

Intermediate interventions to reduce noise



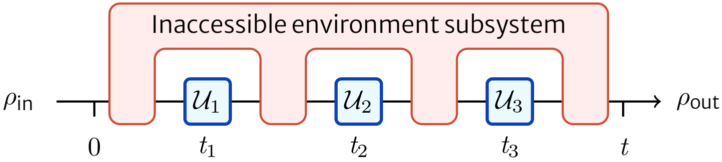
Multi-Time Processes

Model noise as sequence of channels?

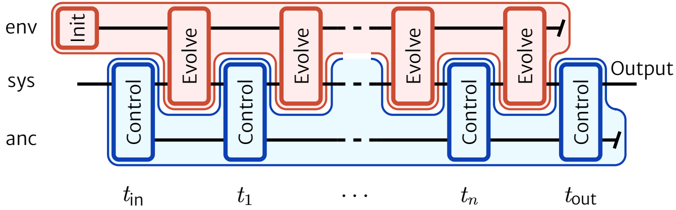


Multi-Time Processes

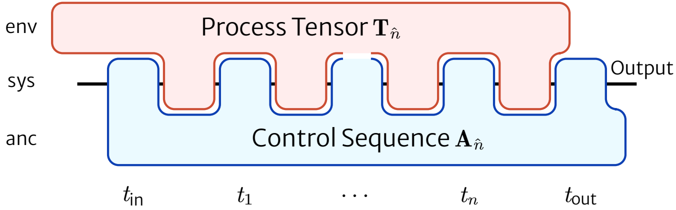
Model noise as a multitime object



Multi-Time Processes



Multi-Time Processes



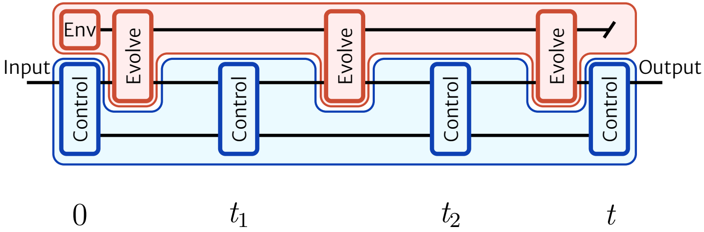
Contracting tensors: $\rho^s_{\text{out}} := [\![\mathbf{T}_{\hat{n}} | \mathbf{A}_{\hat{n}}]\!]$, $\hat{n} = \{t_1, \ldots, t_n\}$

Resource Theories for Noise Reduction

New Kind of Dynamical Resource Theory

- Resource Objects: Process Tensors
- Resource Transformations: ???

Superprocesses



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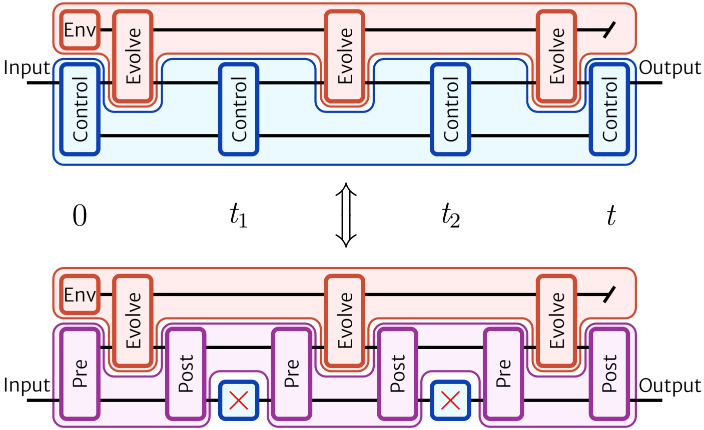
Superprocesses



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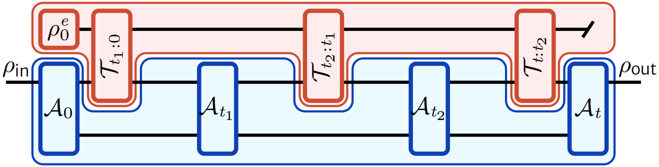
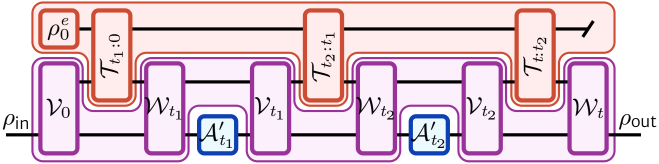
Superprocesses



$$\Updownarrow$$

$$[\![\mathbf{T}_{\hat{n}}|\mathbf{A}_{\hat{n}}]\!] = [\![\mathbf{T}_{\hat{n}}|\mathbf{Z}_{\hat{n}}|\mathbf{A}'_{\hat{n}}]\!]$$

$$\Updownarrow$$



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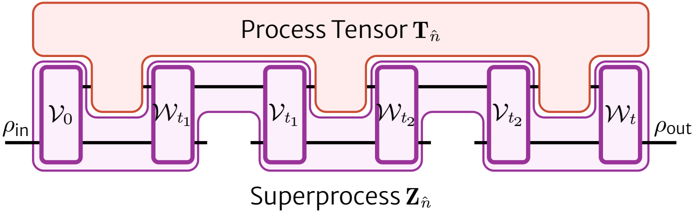
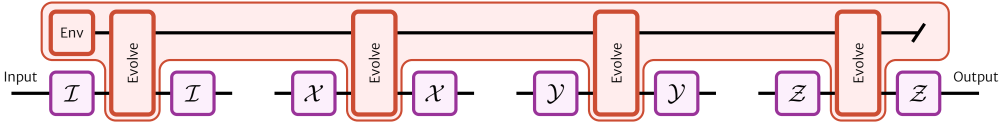
Superprocesses



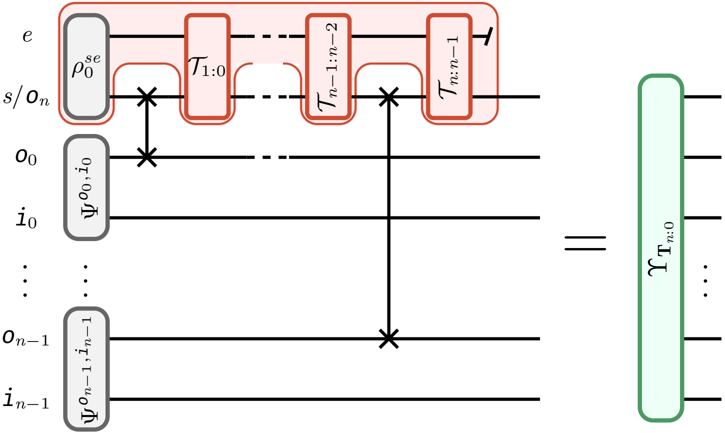
Action of Superprocess: $\mathbf{T}'_{\hat{n}} := [\![\mathbf{T}_{\hat{n}} | \mathbf{Z}_{\hat{n}}$

Dynamical Decoupling as a Superprocess

Dynamical decoupling can be cast as a specific kind of superprocess.



Choi Isomorphism: multitime process ⇔ multipartite state

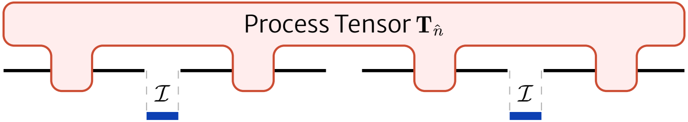


Resource Distillation *in Time*

We can concentrate resources amongst temporal subsystems!

Temporal Coarse-Graining

Second type of resource transformation



Temporal Coarse-Graining $\mathbf{I}_{\hat{n}\setminus\hat{m}}$

Action of Coarse-Graining: $\mathbf{T}_{\hat{m}} := [\![\mathbf{T}_{\hat{n}}|\mathbf{I}_{\hat{n}\setminus\hat{m}}\,, \ \hat{m} \subseteq \hat{n}$

RT for Noise Reduction: Independent Quantum Instruments

Potential Resources: Arbitrary process tensors $\mathbf{T}_{\hat{n}}$.

Free Transformations: $\mathbf{Z}_{\hat{n}}[\mathbf{I}_{\hat{n}\setminus\hat{m}}]$ consist of superprocesses and/or temporal coarse-graining.

Constraints: free superprocesses are temporally local sequences of quantum operations

$$\mathbf{Z}_{\hat{n}} = \mathcal{W}_t \otimes \left(\bigotimes_{i=1}^{n} \mathcal{W}_{t_i} \otimes \mathcal{V}_{t_i} \right) \otimes \mathcal{V}_0 .$$

Monotones of IQI

Total mutual information I is a monotone.

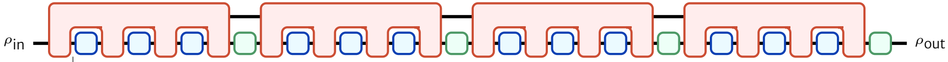
$$I(\mathbf{T}_{\hat{n}}) = S\left(\mathbf{T}_{\hat{n}} \| \mathbf{T}_{\hat{n}}^{\text{marg}}\right) \text{ with } \mathbf{T}^{\text{marg}} := \bigotimes_{k=1}^{2(n+1)} \text{tr}_{\bar{k}}\{\mathbf{T}_{\hat{n}}\},$$

I can be split into two parts: $I(\mathbf{T}_{\hat{n}}) = M(\mathbf{T}_{\hat{n}}) + N(\mathbf{T}_{\hat{n}})$.

IQI is non-convex, but its monotones require no optimisation to compute.

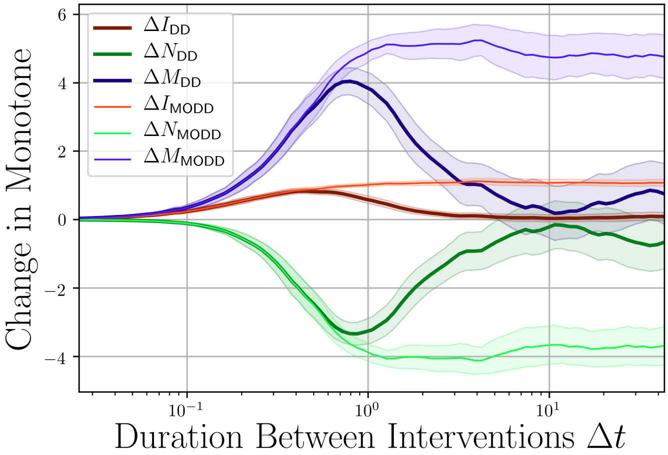
Can We Improve Dynamical Decoupling?

Multitimescale Optimal Dynamical Decoupling (MODD) tailors DD sequences to be applied at every available timescale.



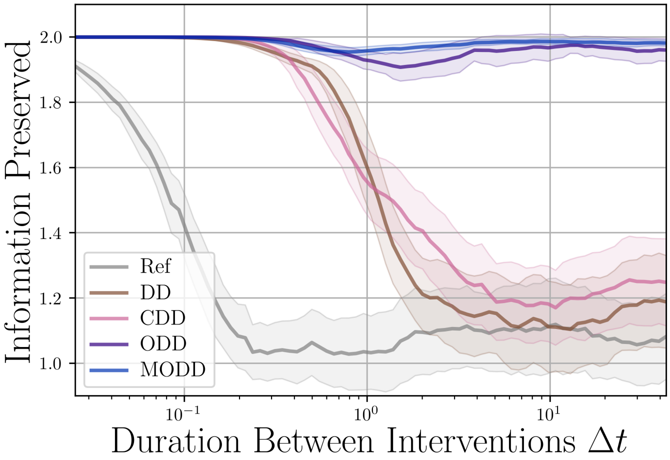
DD	$\mathcal{X}$	$\mathcal{Z}$	$\mathcal{X}$	$\mathcal{Z}$	$\mathcal{X}$	$\mathcal{Z}$	$\mathcal{X}$	$\mathcal{Z}$	$\mathcal{X}$	$\mathcal{Z}$	$\mathcal{X}$	$\mathcal{Z}$	$\mathcal{X}$	$\mathcal{Z}$	$\mathcal{X}$	$\mathcal{Z}$
CDD	$\mathcal{X}$	$\mathcal{Z}$	$\mathcal{X}$	$(\mathcal{Z}\mathcal{X})$	$\mathcal{X}$	$\mathcal{Z}$	$\mathcal{X}$	$(\mathcal{Z}\mathcal{Z})$	$\mathcal{X}$	$\mathcal{Z}$	$\mathcal{X}$	$(\mathcal{Z}\mathcal{X})$	$\mathcal{X}$	$\mathcal{Z}$	$\mathcal{X}$	$(\mathcal{Z}\mathcal{Z})$
Opt.	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$(\mathcal{A}_4\mathcal{B}_1)$	$\mathcal{A}_5$	$\mathcal{A}_6$	$\mathcal{A}_7$	$(\mathcal{A}_8\mathcal{B}_2)$	$\mathcal{A}_9$	$\mathcal{A}_{10}$	$\mathcal{A}_{11}$	$(\mathcal{A}_{12}\mathcal{B}_3)$	$\mathcal{A}_{13}$	$\mathcal{A}_{14}$	$\mathcal{A}_{15}$	$(\mathcal{A}_{16}\mathcal{B}_4)$

Conversion of Non-Markovianity



G. D. Berk, S. Milz, F. A.Pollock, and K. Modi, **arXiv**:2110.02613 **(2021)**.

Preservation of Information at the Channel-Level



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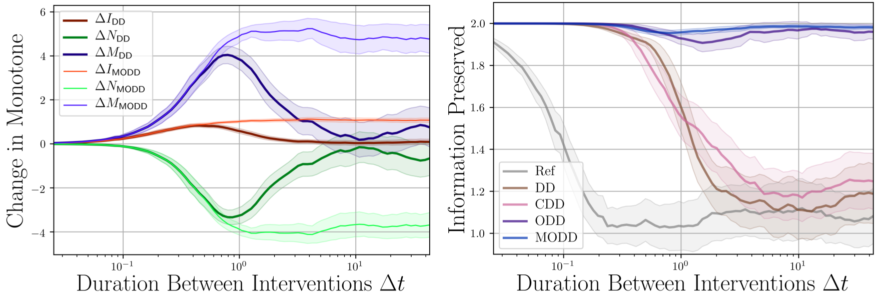

Convex RT: Entanglement Breaking Quantum Instruments

Free superprocesses of IQI:

$$\mathbf{Z}_{\hat{n}} = \mathcal{W}_t \otimes \left(\bigotimes_{i=1}^{n} \mathcal{W}_{t_i} \otimes \mathcal{V}_{t_i} \right) \otimes \mathcal{V}_0$$

Convex combinations of trace non-increasing combs:

$$\mathbf{Z}_{\hat{n}} = \sum_k p_k \mathcal{W}_{t_k} \otimes \left(\bigotimes_{i=1}^{n} \mathcal{W}_{t_{i_k}} \otimes \mathcal{V}_{t_{i_k}} \right) \otimes \mathcal{V}_{0_k}$$

Entanglement in time is the resource in EBQI.

Bounding Noise Reduction

Define another theory $\mathrm{ARNG}_{\mathrm{EBQI}}$ using free superprocesses that are asymptotically resource non-generating w.r.t. the free processes of EBQI.

Bound on noise reduction:

$$r(\mathbf{T}_{\hat{n}} \to \mathbf{T}'_{\hat{n}}) \leq \frac{S^{\infty}_{\mathsf{RF}}(\mathbf{T}_{\hat{n}})}{S^{\infty}_{\mathsf{RF}}(\mathbf{T}'_{\hat{n}})},$$

Issues with tightness because permutations of temporally separated subsystems are disallowed.

Summary

1. Noise Reduction as a Resource Transformation
2. Multitime Processes as Resources
3. Resource Theories for Noise Reduction
4. Numerical Results
5. Bounding Noise Reduction