METROLOGICAL RESOURCES

BRACED FOR THE WORST

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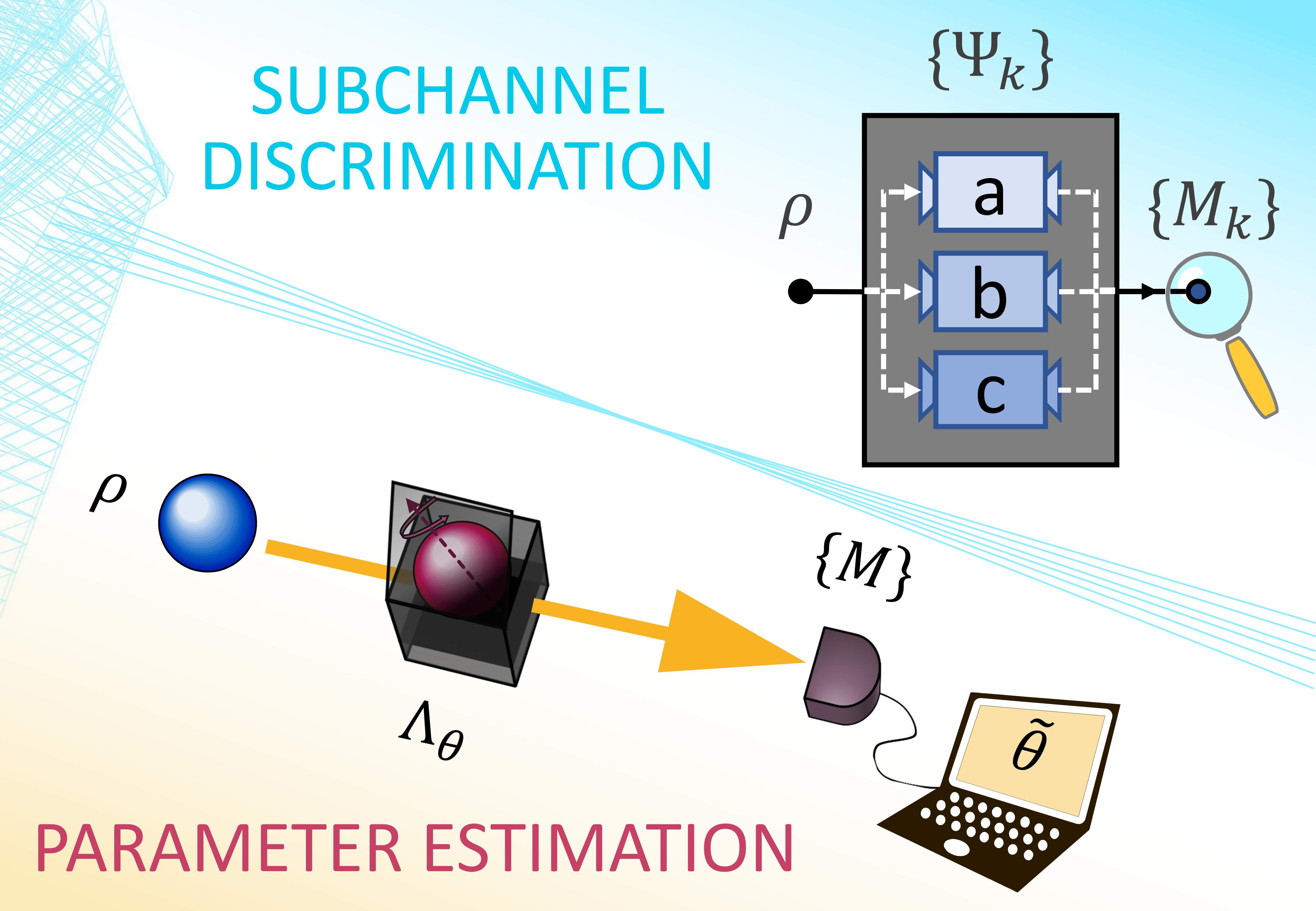
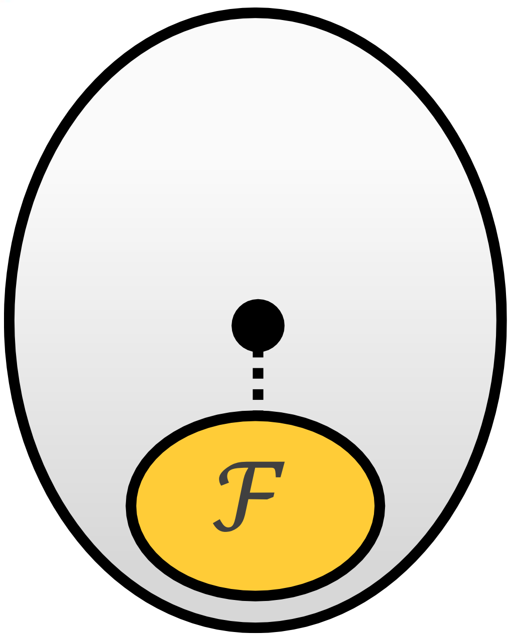
QUANTUM METROLOGY

- exploits quantum mechanical features
- to improve the available precision
- in measuring physical parameters



SUBCHANNEL
DISCRIMINATION
$\{\Psi_k\}$
ρ
a
b
c
$\{M_k\}$
ρ
Λ_θ
$\{M\}$
$\tilde{\theta}$
PARAMETER ESTIMATION

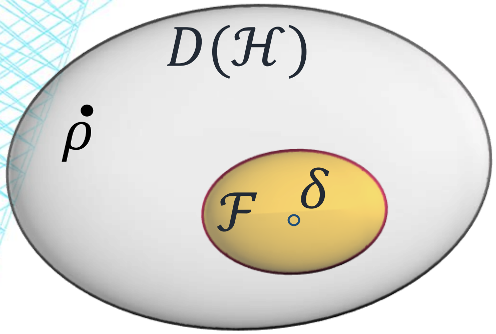
(CONVEX) QUANTUM RESOURCES

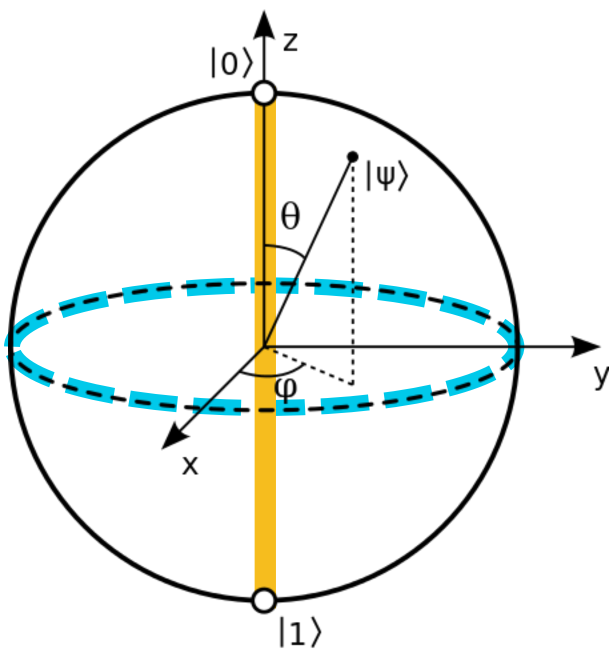


The set $\mathcal{F}$ of free states is

- **Convex**
  (mixing and forgetting does not create any resource)
- **Closed**
  (the limit of a sequence of free states is a free state)

RESOURCE THEORY OF COHERENCE





Free states

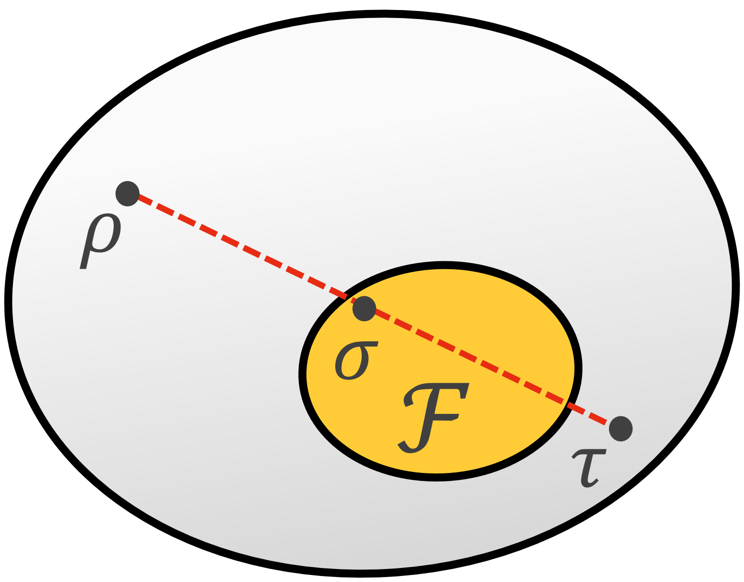
- **Incoherent states:** States diagonal in a chosen reference basis $\{|j\rangle\}$: $\delta \in \mathcal{F}$: $\delta = \sum_j p_j |j\rangle\langle j|$, or equivalently $\delta = \Delta(\delta)$ with $\Delta(\rho) = \sum_j |j\rangle\langle j|\rho\,|j\rangle\langle j|$
- E.g. for one qubit, with respect to the computational basis, the states $|0\rangle$ and $|1\rangle$ and their mixtures $p\,|0\rangle\langle 0| + (1-p)|1\rangle\langle 1|$ are incoherent (free); conversely, any equatorial state, i.e. $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, is a maximally coherent state.

Free operations

- Operations $\mathcal{O}$ unable to create coherence, that map incoherent states into incoherent states (e.g. MIO, DIO, IO, SIO, …)

ROBUSTNESS OF A RESOURCE

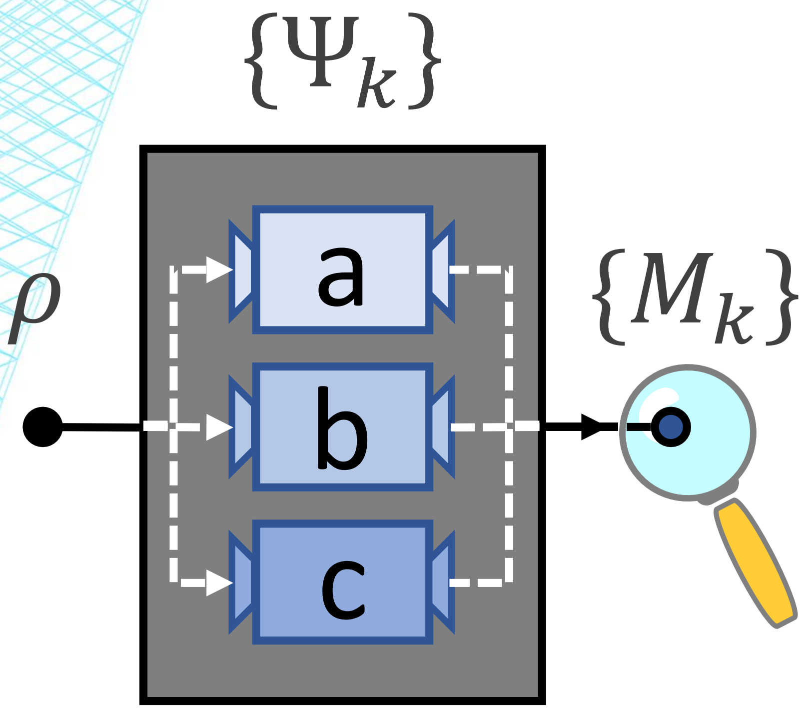
$$R_{\mathcal{F}}(\rho) = \min_{\tau} \left\{ s \geq 0 \mid \frac{\rho + s\,\tau}{1+s} =: \sigma \in \mathcal{F} \right\}$$



Convex optimisation

- minimise $\mathrm{Tr}[\rho X] - 1$
- subject to
  - $X \geq 0$
  - $\mathrm{Tr}[\sigma X] \leq 1 \;\forall\, \sigma \in \mathcal{F}$

SUBCHANNEL DISCRIMINATION

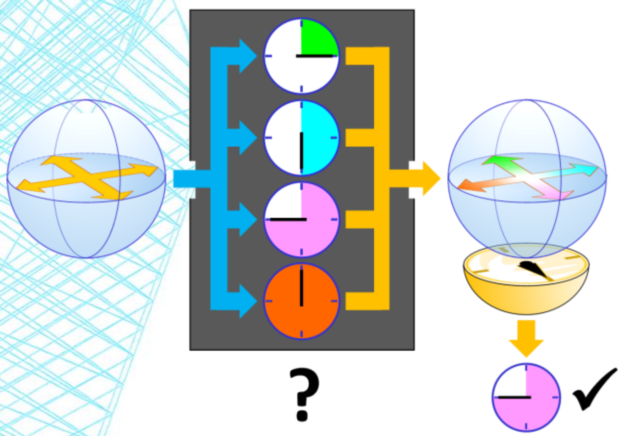


Goal of the game: maximise the probability of success

$$p_{succ}(\rho, \{\Psi_k\}, \{M_k\}) = \sum_k \mathrm{Tr}[M_k \Psi_k(\rho)]$$

- Notice: this includes channel discrimination (with $\Psi_k = p_k \Lambda_k$)

$$U_k = e^{-i\phi_k G}$$



PRL **116,** 150502 (2016) PHYSICAL REVIEW LETTERS week ending 15 APRIL 2016

Robustness of Coherence: An Operational and Observable Measure of Quantum Coherence

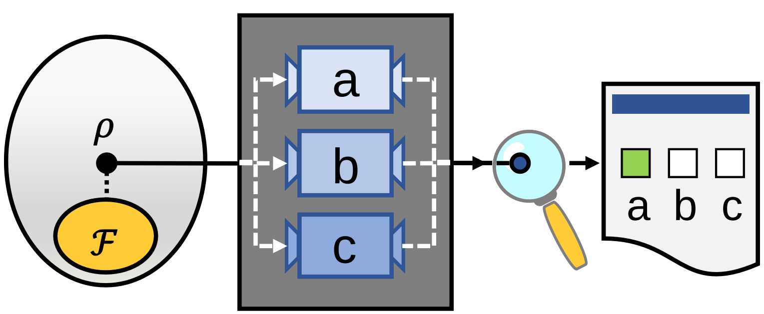
Carmine Napoli,[1,2] Thomas R. Bromley,[2] Marco Cianciaruso,[1,2] Marco Piani,[3] Nathaniel Johnston,[4] and Gerardo Adesso[2]

PHYSICAL REVIEW LETTERS **122,** 140402 (2019)

Editors' Suggestion

Operational Advantage of Quantum Resources in Subchannel Discrimination

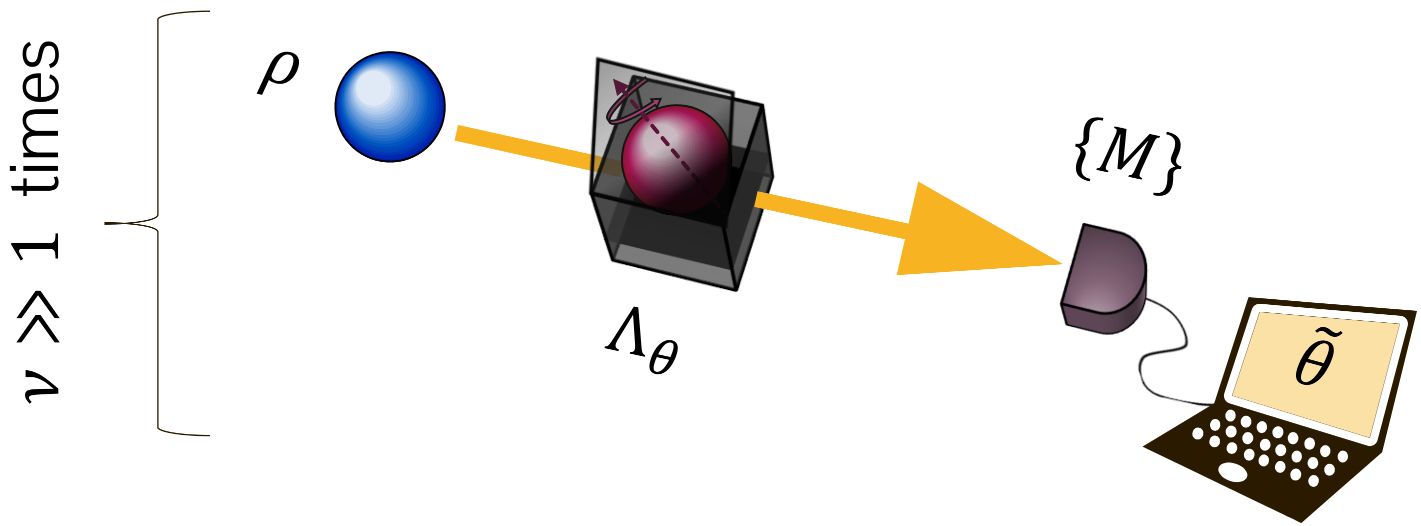
Ryuji Takagi,[1,*] Bartosz Regula,[2,3,4,†] Kaifeng Bu,[5,6,‡] Zi-Wen Liu,[7,1,§] and Gerardo Adesso[2,||]

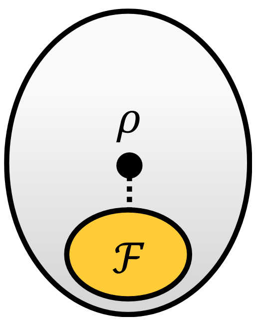


In any convex resource theory, for every ρ

$$\max_{\{\Psi_k\},\{M_k\}} \frac{p_{succ}(\rho,\{\Psi_k\},\{M_k\})}{\max_{\sigma\in\mathcal{F}} p_{succ}(\sigma,\{\Psi_k\},\{M_k\})} = 1 + R_{\mathcal{F}}(\rho)$$

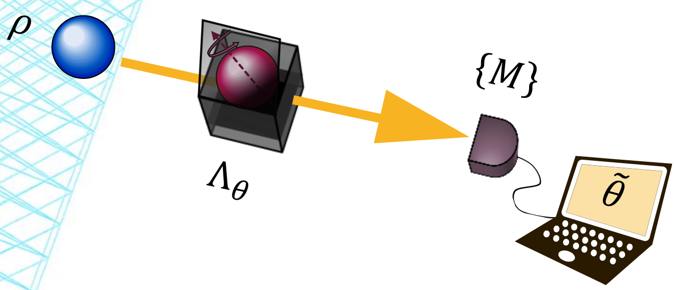
PARAMETER ESTIMATION



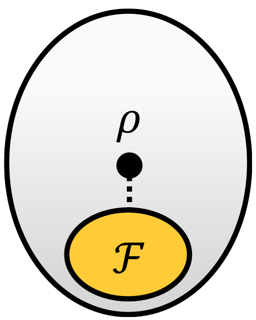


- Quantum Cramér-Rao bound: the estimation error satisfies $\Delta\theta^2 \geq (\nu\, H)^{-1}$, where H is the **quantum Fisher information**
- Define metrological advantage: $N_Q(\rho) = Q(\rho) - \max_{\sigma\in\mathcal{F}} Q(\sigma)$

PARAMETER ESTIMATION



$$N_Q(\rho) = Q(\rho) - \max_{\sigma \in \mathcal{F}} Q(\sigma)$$



PHYSICAL REVIEW X **10**, 041012 (2020)

Entanglement between Identical Particles Is a Useful and Consistent Resource

Benjamin Morris,[1,*,†] Benjamin Yadin,[1,2,*,‡] Matteo Fadel,[3,4] Tilman Zibold,[3]
Philipp Treutlein,[3] and Gerardo Adesso[1,§]

PHYSICAL REVIEW LETTERS **127**, 200402 (2021)

Fisher Information Universally Identifies Quantum Resources

Kok Chuan Tan,[*] Varun Narasimhachar, and Bartosz Regula
School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore 637371, Republic of Singapore

Theorem 1. There exists a parameter estimation problem with quantum channel Φ_θ and measurement M that satisfies $N_C(\rho|M) > 0$ and $N_Q(\rho) > 0$ if and only if $\rho \notin \mathcal{F}$.

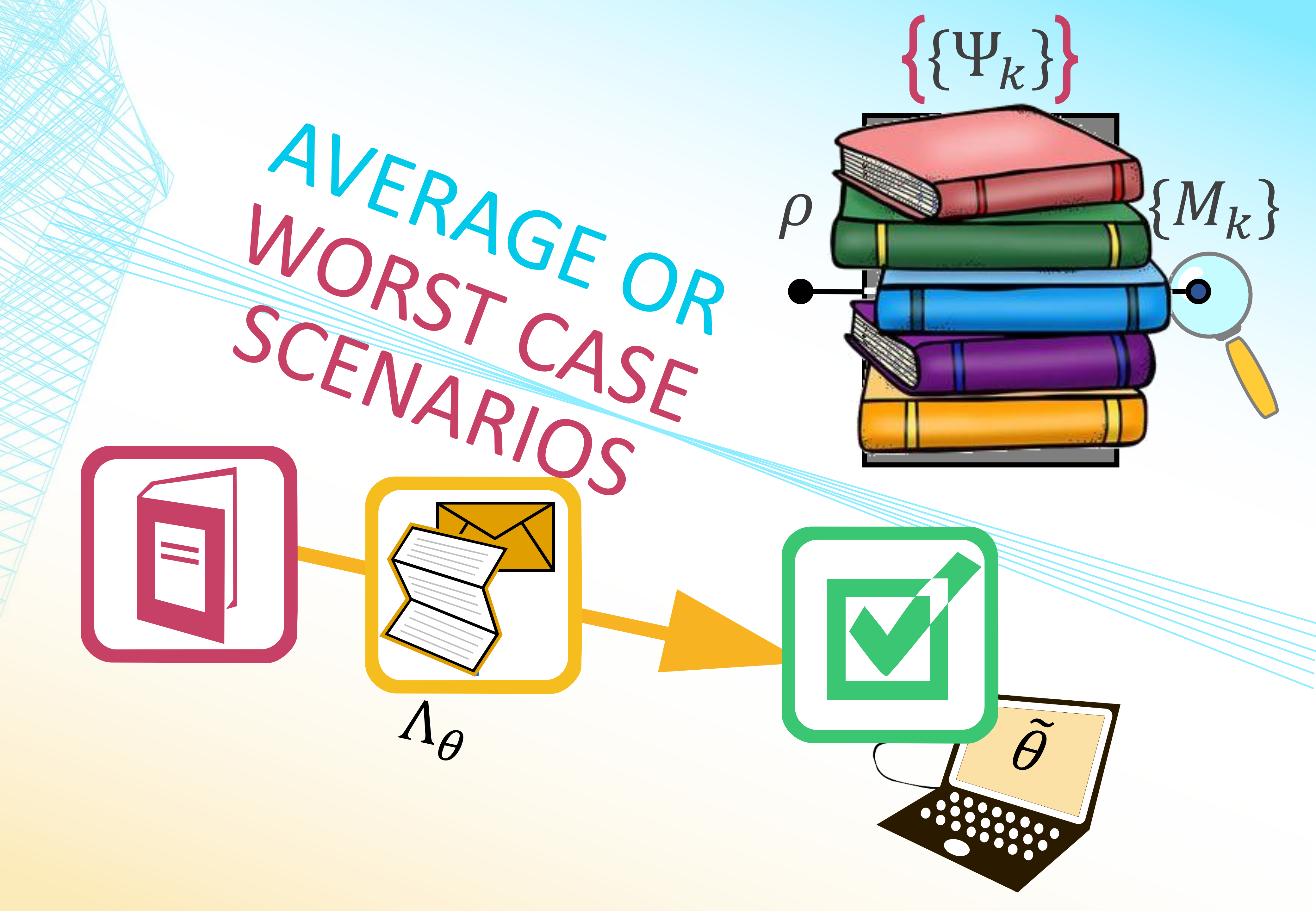
Quantum resources

from mathematical foundations to operational characterisation

Singapore

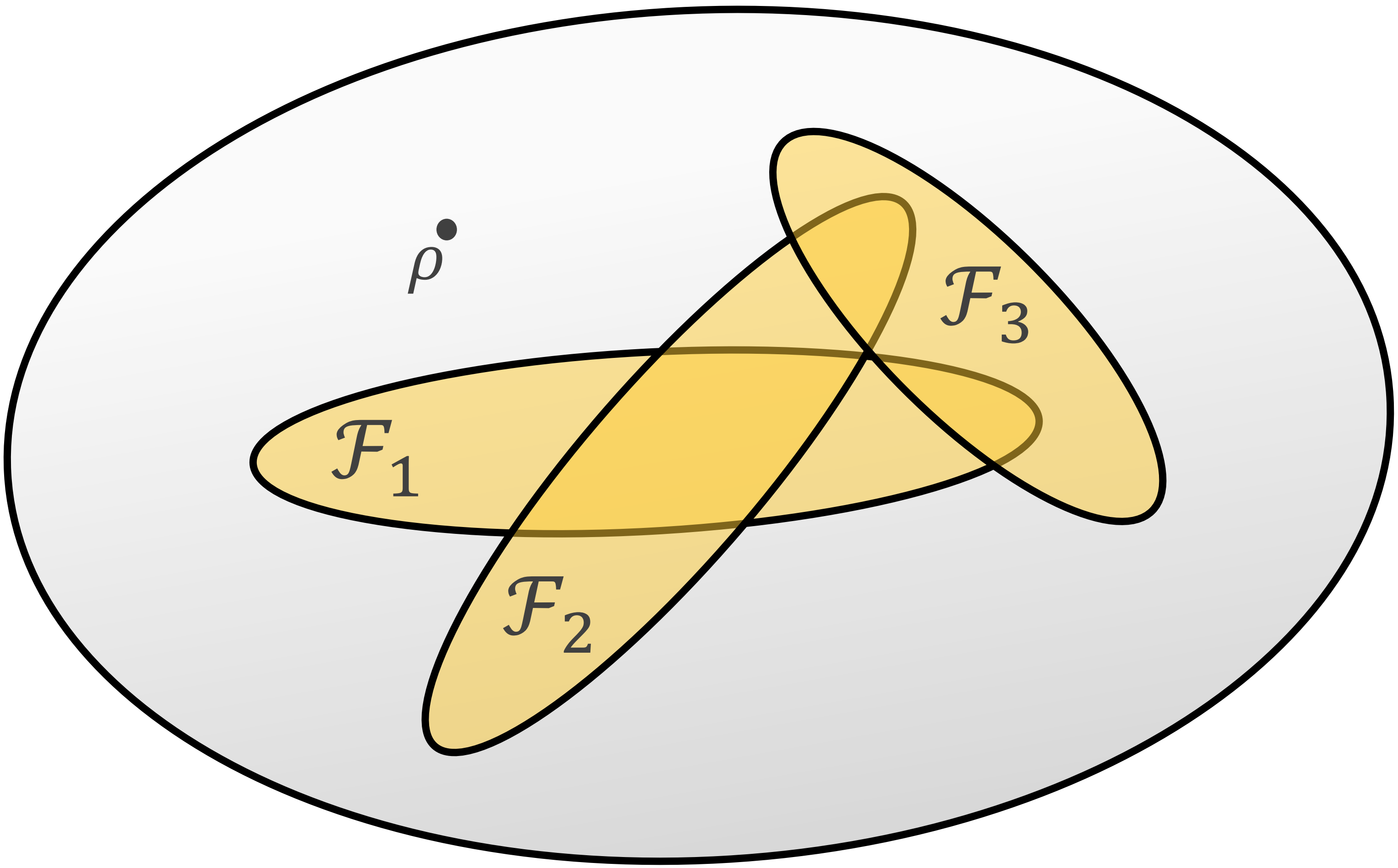
December 2022

Everybody
is a genius. But if you a judge a fish by its ability to climb a tree, it will live its whole lfe believing it is stupid

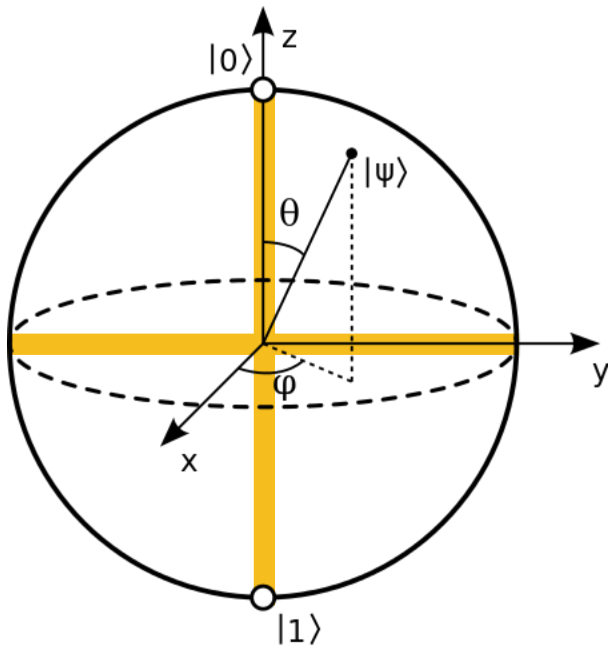

AVERAGE OR
WORST CASE
SCENARIOS
$\{\{\Psi_k\}\}$
ρ
$\{M_k\}$
Λ_θ
$\tilde{\theta}$

NON CONVEX RESOURCES

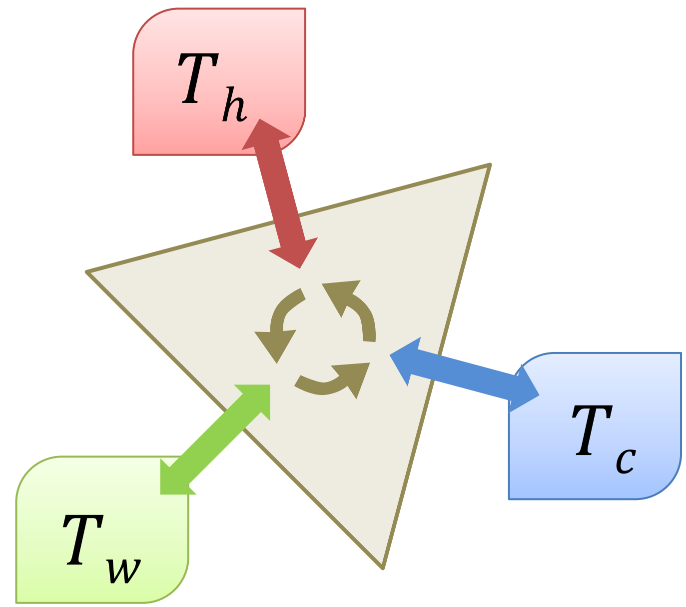
Free states: $\mathcal{F} = \cup_j \mathcal{F}_j$



EXAMPLES



- Thermodynamics with thermal baths at different temperatures (resource engines)

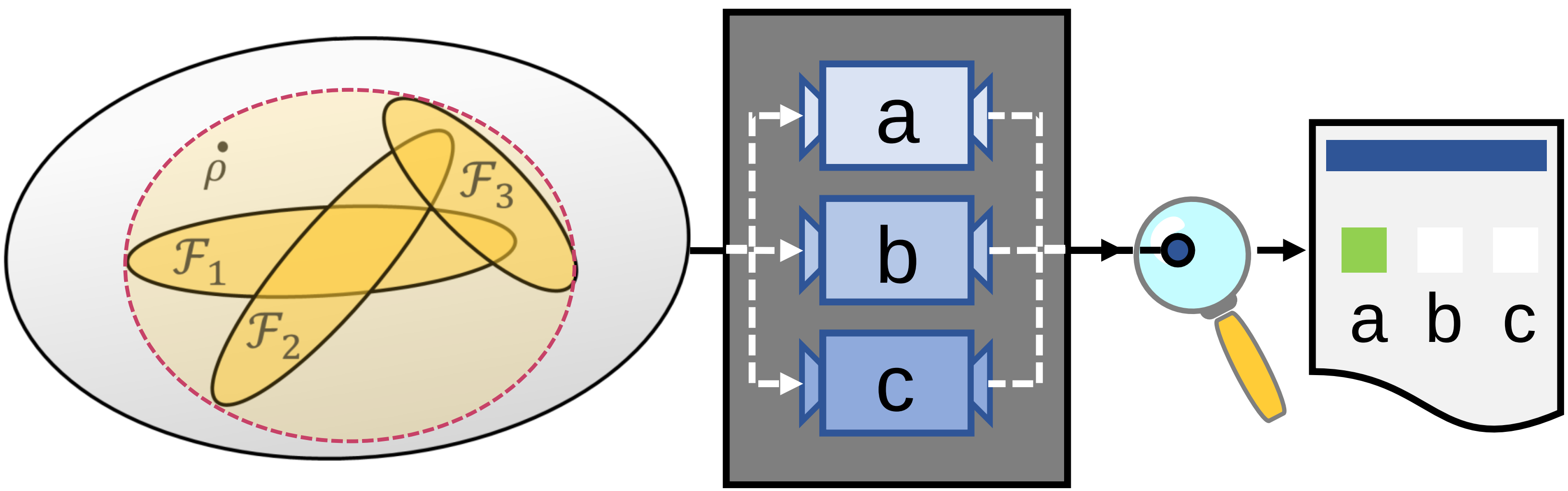


- Quantum coherence in multiple reference bases (versatile sensors)

SUBCHANNEL DISCRIMINATION

- The advantage w.r.t non-convex set $\mathcal{F} = \bigcup_j \mathcal{F}_j$ amounts to robustness w.r.t. the convex hull of $\mathcal{F}$

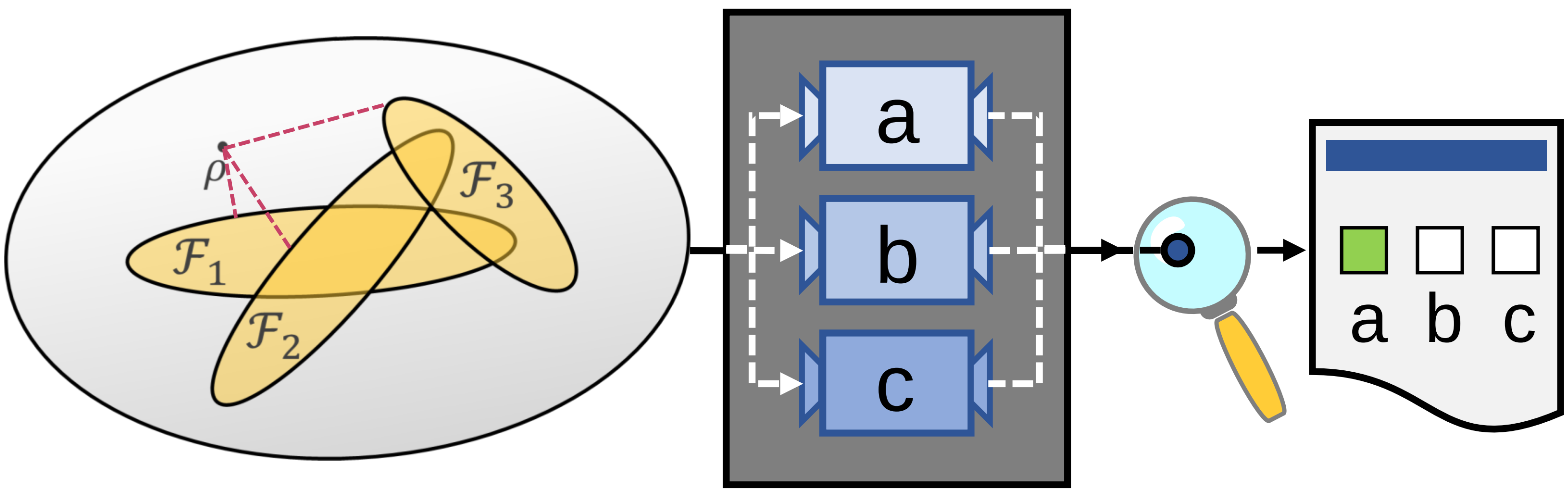
$$\max_{\{\Psi_k\},\{M_k\}} \frac{p_{succ}(\rho,\{\Psi_k\},\{M_k\})}{\max_{\sigma\in\mathcal{F}} p_{succ}(\sigma,\{\Psi_k\},\{M_k\})} = 1 + R_{\mathrm{conv}(\mathcal{F})}(\rho)$$



WORST CASE DISCRIMINATION

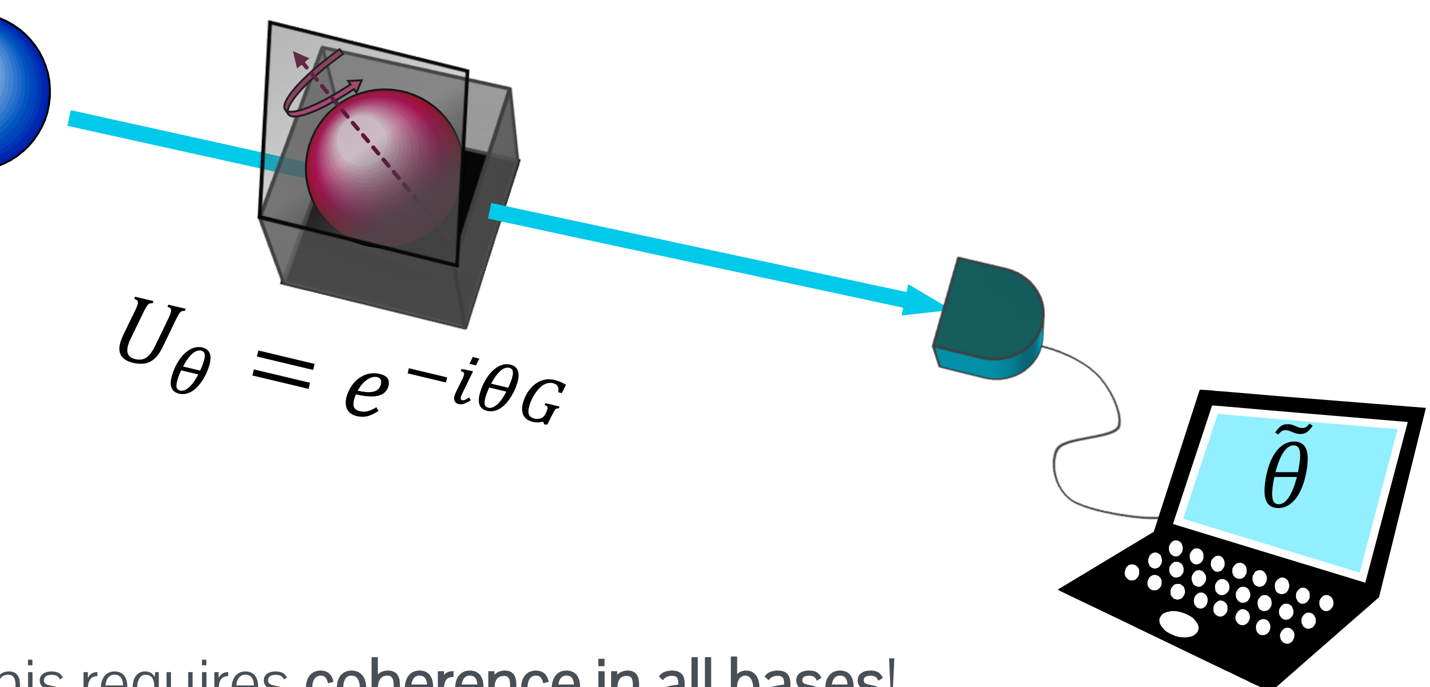
- The **minimax advantage** w.r.t. each subset $\mathcal{F}_j$ amounts to **robustness** w.r.t. the set $\mathcal{F} = \cup_j \mathcal{F}_j$

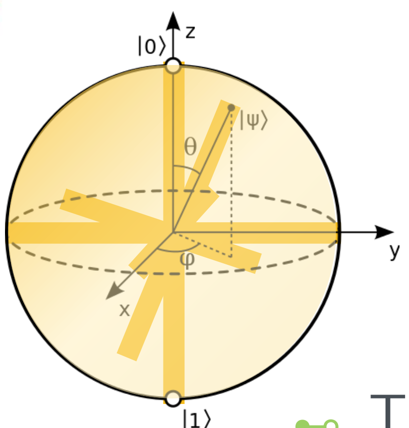
$$\inf_j \max_{\{\Psi_k\},\{M_k\}} \frac{p_{succ}(\rho,\{\Psi_k\},\{M_k\})}{\max_{\sigma\in\mathcal{F}_j} p_{succ}(\sigma,\{\Psi_k\},\{M_k\})} = 1 + \inf_j R_{\mathcal{F}_j}(\rho) = 1 + R_{\mathcal{F}}(\rho)$$



BLIND PHASE ESTIMATION

- Only the spectrum of the generator G known a priori
- Eigenbasis (non-degenerate) revealed after preparation
- Worst-case scenario: minimum quantum Fisher Info Q

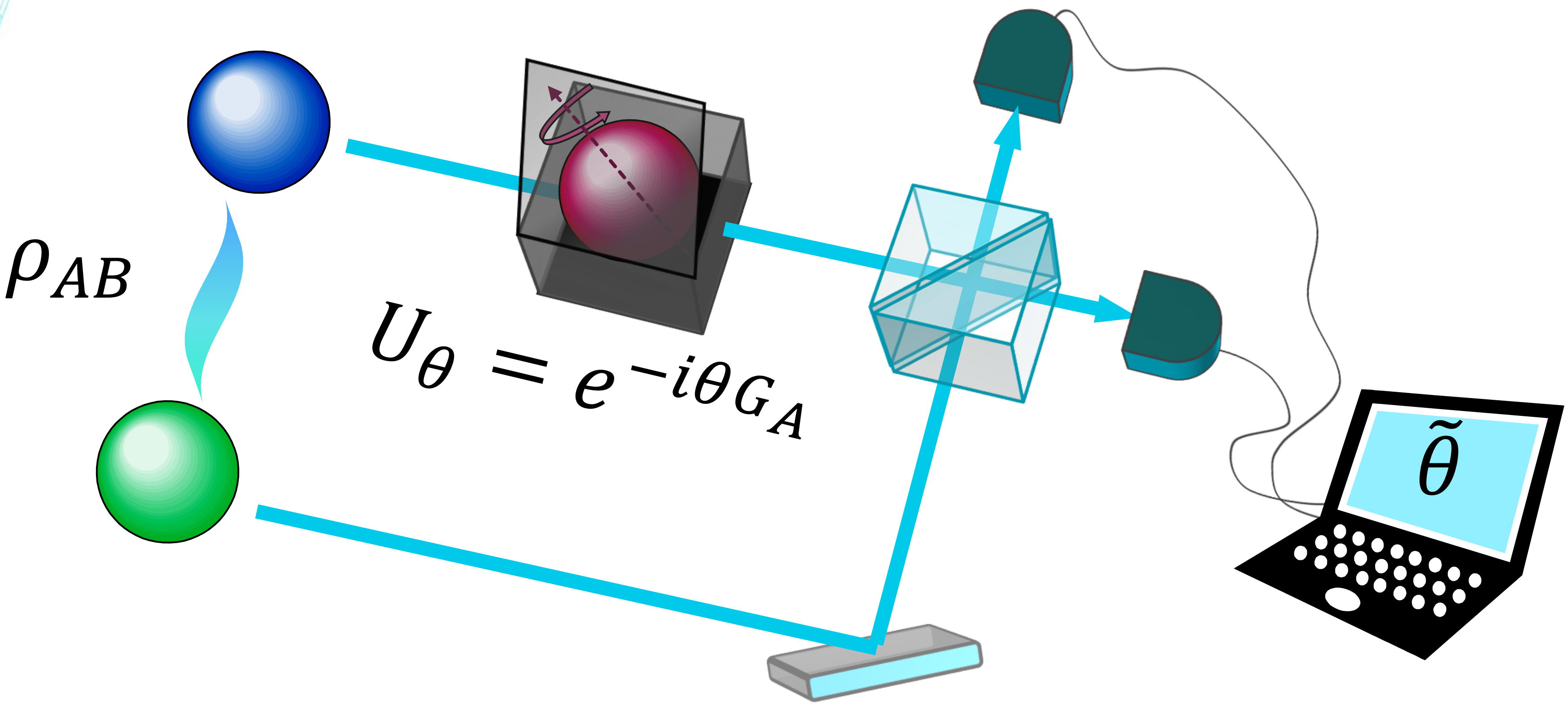




- This requires **coherence in all bases**!

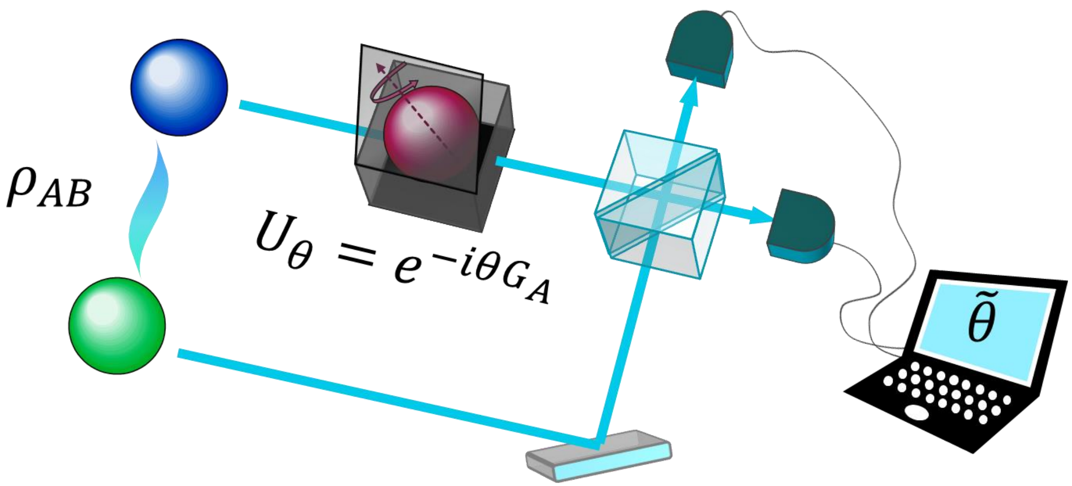
INTERFEROMETRIC POWER

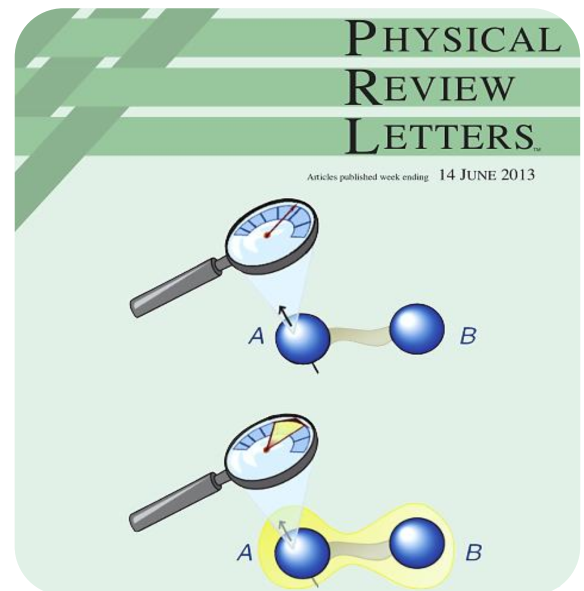
- Worst-case scenario: minimum quantum Fisher Info Q
- $P(\rho_{AB}) = \frac{1}{4} \inf_{G_A} Q(\rho_{AB}\,; G_A)$ for a bipartite probe ρ_{AB}



INTERFEROMETRIC POWER

- A measure of **quantum discordant correlations**
- $P(\rho_{AB}) = 0$ if and only if $\rho_{AB} = \sum_i p_i |i\rangle\langle i|_A \otimes \tau_{i\,B}$





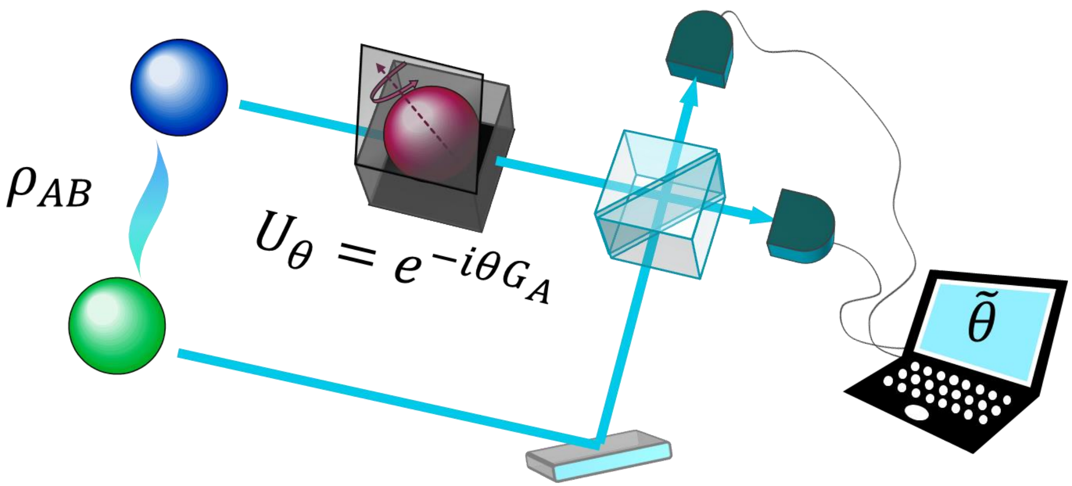
PRL **112,** 210401 (2014) PHYSICAL REVIEW LETTERS

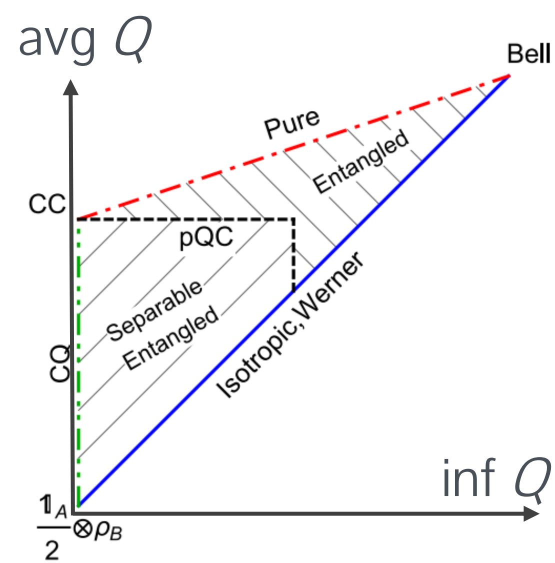
Quantum Discord Determines the Interferometric Power of Quantum States

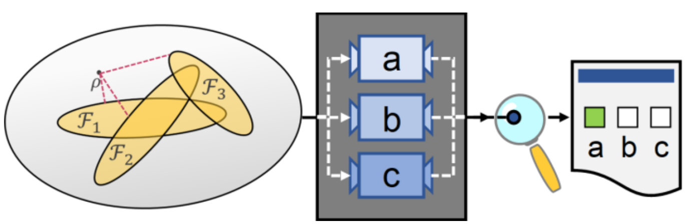
Davide Girolami,[1,2,8] Alexandre M. Souza,[3] Vittorio Giovannetti,[4] Tommaso Tufarelli,[5] Jefferson G. Filgueiras,[6] Roberto S. Sarthour,[3] Diogo O. Soares-Pinto,[7] Ivan S. Oliveira,[3] and Gerardo Adesso[1,*]

WHAT ABOUT THE AVERAGE?

- $\langle Q(\rho_{AB}; G_A)\rangle_{\mathcal{U}(G_A)} = 0$ if and only if $\rho_{AB} = \frac{\mathbb{I}_A}{d_A} \otimes \tau_B$
- Either local purity or correlations would be resources







Average for subchannel discrimination?

- OPEN PROBLEM

SUMMARY

- Convex resource theories
  - Every resource is useful for discrimination or estimation
- Non-convex theories
  - Worst case discrimination advantage = robustness
  - Worst case estimation = interferometric power (local coherence = discord)

OUTLOOK

- Hybrid resource theories
  - Worst case for hybrid state/measurements
- Examples and applications
  - Quantum thermodynamics
  - Magic state distillation
  - Experimentally restricted settings and operations
  - Average figures of merit


THANK YOU